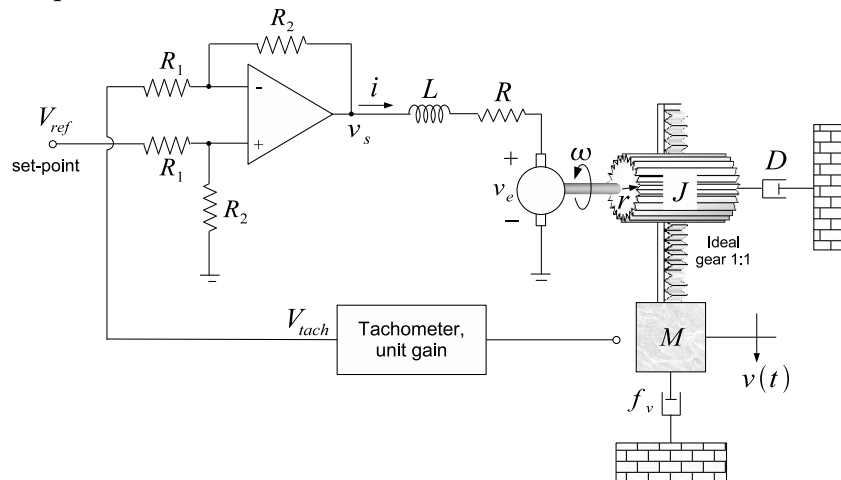Problem Set #5

Posted: Friday, Oct. 12, '07   <u>Due:</u> **Friday, Oct. 19, '07**

In this Problem Set, we will practice using MATLAB to model and analyze the response of dynamical systems. MIT supports MATLAB in the Athena clusters, the Mechanical Engineering computer clusters, and other locations (e.g., most research labs.) You may also use the educational version of the software, for which you qualify as long as you are a registered student at MIT.

The systems we'll be analyzing are based on a modified version of the DC motor system that we analyzed in Lecture 12, with pinion–rack load and velocity feedback. In the lecture, we neglected the motor's inductance L; here, we will analyze L's effect on the open–loop and closed–loop system responses. We will walk you step–by–step through the modeling and analysis of the open–loop and closed–loop systems and show you how to find out the effects of changing the gain in the feedback loop analytically and with MATLAB .

The references to equations are from the document labeled "Supplement to Lecture 12," ("Supplement" for short) which you can download from the **MIT Server.**
You should study the Supplement thoroughly and make sure you understand it well before attempting this problem set. You will also need the lecture notes from Lecture 15, which are excerpts from MATLAB 's LTI Toolbox tutorial.

The model is shown above, where the DC motor's inductance L has been included in the circuit diagram. Since we've added one energy storage element, we expect to be dealing with a second–order system in this case. Whether it is overdamped or underdamped will depend on the relative strengths of the energy storage components

(inductor, inertias) and the dissipation elements (resistor, dampers.)

1. Modify the Laplace–domain KVL equation (2) to include the inductance. Argue that (4), (6), (9–10) need not be modified in this system, and thus (11) remains valid.

2. Substitute (11) to your new KVL equation from question 1 and derive the open–loop transfer function $V(s)/V_s(s)$.

3. Substitute $L = 1$ H and the remaining numerical values from the Supplement, and find the poles of your transfer function. You should find that this system is *underdamped.*

4. Use the LTI tool to plot the step response of the open–loop system. Compute the steady–state analytically using the transfer function from question 3 and verify that it matches the LTI tool's prediction.

5. Use the plot from question 4 to estimate the rise time of the system. How does it compare with the rise time of the open–loop system from the Supplement (*i.e.,* the identical system but without the inductance.)

6. Using the numerical form of the open–loop transfer function from question 3, derive the closed–loop transfer function with the feedback gain K as a variable.

7. Compute the steady–state velocity and the steady–state error as function of feedback gain K for the closed–loop system of question 6. Use an argument based on the *physics* of the system to explain similarities or differences to the inductance–free system that was analyzed in the Supplement.

8. Substitute $K = 1$ into the closed–loop transfer function from question 6 and use Nise section 4.6 to estimate the peak time, percent overshoot and settling time.

9. Use the LTI tool to plot the step response of the closed loop system with $K = 1$ and verify that the steady–state velocity, peak time, percent overshoot and settling time that you obtain from the numerical simulation are in agreement with the analytical estimates.

10. Substitute $K = 25$ into the closed–loop transfer function from question 6, and use the LTI tool to plot the step response of this high–gain system. Comment, based on the plot and as thoroughly as you can, on the similarities and differences, if any, to the low–gain system step response from questions 8–9.

11. Now go back to your open–loop transfer function from question 2 and substitute $L = 1/16 = 0.0625$ H. Find the poles of the new transfer function; you should observe that this system is *overdamped.* Comment on the locations of the two poles relative to the pole of the inductance–free system from the Supplement.

12. Using the numerical form of the open–loop transfer function from question 11,

derive the closed–loop transfer function with the feedback gain K as a variable.

13. If the feedback gain K were to vary from 0 to $+\infty$, would the system be **(i)** always overdamped; **(ii)** always underdamped; **(iii)** transition to/from overdamped from/to underdamped? Justify your answer using the transfer function you derived in question 12.

14. Compute the steady–state velocity and the steady–state error as function of feedback gain K for the closed–loop system of question 12. Are they the same or different than those of the high-inductance closed–loop system from question 7? Is that consistent with the physics–based argument that you made in your answer to question 7?

15. Use the LTI tool to plot the step response of the closed–loop system from question 12 for gains $K = 1$ and $K = 25$. Verify that the responses generated by the LTI tool are consistent with your answer to questions 13, 14.