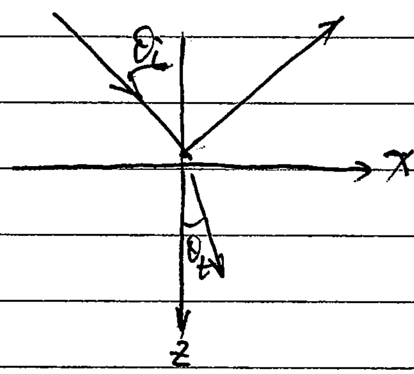
Review of lecture 8



$$\vec{E}_i = \vec{E}_{i\parallel} \exp[-i(\omega t - k_{xi}x - k_{zi}z)]$$

$$\vec{E}_r = \vec{E}_{r\parallel} \exp[-i(\omega t - k_{xr}x - k_{zr}\cdot z)]$$

$$\vec{E}_t = \vec{E}_{t\parallel} \exp[-i(\omega t - k_{xt}x - k_{zt}\cdot z)]$$

Snell law

$$n_1 \sin\theta_i = n_2 \sin\theta_t$$

— momentum conservati.

Fresnel Coefficent / Reflectivity & transmissivity

$$r_\parallel = \frac{n_1\cos\theta_t - n_2\cos\theta_i}{n_1\cos\theta_t + n_2\cos\theta_i} \qquad R = |r|^2$$

$$t_\parallel = \dots \qquad T = \frac{n_2\cos\theta_t}{n_1\cos\theta_i}|t|^2$$

normal $\quad R = \left|\frac{n_1-n_2}{n_1+n_2}\right|^2.$

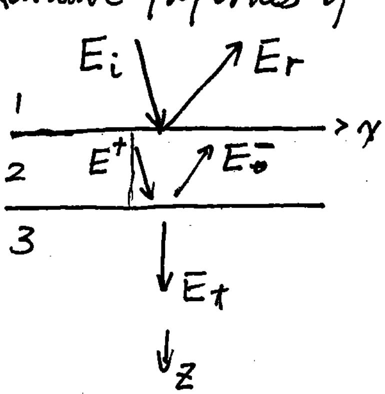
(5) N_1 / N_2

Fresnel formula still valid — 9/2

But $\langle \hat{S}_{tz} \rangle \neq \langle S_{iz} \rangle - \langle S_{rz} \rangle$

there is a surface absorption term.

* Radiative Properties of Thin films



$$\vec{E}^+ = \vec{E}_0^+ \exp[i\vec{k}^+ \cdot \vec{r}]$$

$$\vec{E}^- = \vec{E}_0^- \exp[-i\vec{k}^- \cdot \vec{r}]$$

$$\vec{k}^+ = (\sin\theta_2 \cdot \hat{x} + \cos\theta_2 \cdot \hat{z}) \frac{N_2 \omega}{c_0}$$

$$\vec{k}^- = (\sin\theta_2 \cdot \hat{x} - \cos\theta_2 \, \hat{z}) \frac{N_2 \omega}{c_0}$$

⇒ Interface 1

$$E_{i\parallel}\cos\theta_i + E_{r\parallel}\cos\theta_{i\parallel} = E^+_{\parallel}\cos\theta_2 + E^-_{\parallel}\cos\theta_2 \qquad \boxed{n_1 \sin\theta_i = N_2 \sin\theta_2}$$

$$n_1 E_{i\parallel} - n_1 E_{r\parallel} = N_2 E^+_{\parallel} - N_2 E^-_{\parallel}$$

⇒ Interface 2

$$E^+_{\parallel}\cos\theta_2 \exp(i\cos\theta_2 \cdot d\, N_2\omega/c_0)$$
$$+ E^-_{\parallel}\cos\theta_2 \exp(-i\cos\theta_2\, d\, N_2\omega/c_0) = E_{t\parallel}\cos\theta_t \exp(i\cos\theta_3\, d\, N_3\omega/c_0)$$

$$n_2 E^+_{\parallel} \exp(i\cos\theta_2\, d\, N_2\omega/c_0) - n_2 E^-_{\parallel} \exp(-i\cos\theta_2\, d\, N_2\omega/c_0) = n_3 E_{t\parallel} \exp(i\cos\theta_3 \frac{d\, \omega}{c_0}$$

9/3

$$E_{t\parallel}(d) = E_{t\parallel}\exp\left(i\cos\theta_3\, d\, N_3\,\omega/c_0\right)$$

$$r_\parallel = \frac{E_{r\parallel}}{E_{i\parallel}} = \frac{r_{12\parallel} + r_{23\parallel}e^{2i\varphi_2}}{1 + r_{12\parallel}r_{23\parallel}e^{2i\varphi_2}} \qquad \varphi_2 = \frac{\cos\theta_2\, d\, N_2\,\omega}{c_0}$$

$r_{12\parallel}$, $r_{23\parallel}$ are Fresnel reflection coefficient at 12 & 23

$$t_\parallel = \frac{E_{t\parallel}(d)}{E_{i\parallel}} = \frac{t_{12}t_{23}e^{i\varphi_2}}{1+r_{12}r_{23}e^{2i\varphi_2}}. \quad \overset{\text{if } n_1 - n_3 \text{ real}}{=} \quad \frac{r_{12}^2 + r_{23}^2 + 2r_{12}r_{23}\cos 2\varphi_2}{1 + 2r_{12}r_{23}\cos 2\varphi_2 + r_{12}^2 r_{23}^2}$$

$$R = |r_\parallel|^2, \qquad T_\parallel = \frac{n_3\cos\theta_t}{n_1\cos\theta_i}\,|t_\parallel|^2$$

Discussion: (1) Interference effects.

$$\frac{4\pi n_2 d\cos\theta_2}{\lambda_0} = m\pi.$$

$$d\cos\theta_2 = \frac{m\lambda_0}{4n_2}$$

normal incidence

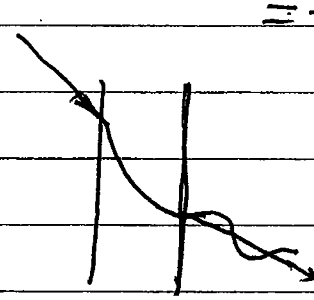
m odd ϕ $$R = \left(\frac{r_{12}-r_{23}}{1-r_{12}r_{23}}\right)^2 = \left(\frac{n_1n_3 - n_2^2}{n_1n_3+n_2^2}\right)^2$$

m even $$R = \left(\frac{r_{12}+r_{23}}{1+r_{12}r_{23}}\right)^2 = \left(\frac{n_1-n_3}{n_1+n_3}\right)^2$$

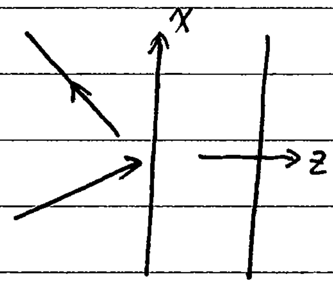
(2) tunneling when $\theta_i > \theta_{cr}$

$$n_2\cos\theta_2 = n_2\sqrt{1-\left(\frac{n_1\sin\theta_1}{n_2}\right)^2}$$
$$= -ib$$

$T_\parallel \neq 0$



Multilayer of Thin Films



$$\begin{pmatrix} E_x(z) \\ H_y(z) \end{pmatrix} = \begin{pmatrix} \cos\varphi(z) & ip_2 \sin\varphi(z) \\ \frac{i}{p_2}\sin\varphi(z) & \cos\varphi(z) \end{pmatrix} \begin{pmatrix} E_x(0) \\ H_y(0) \end{pmatrix}$$

Transfer matrix M^{-1}

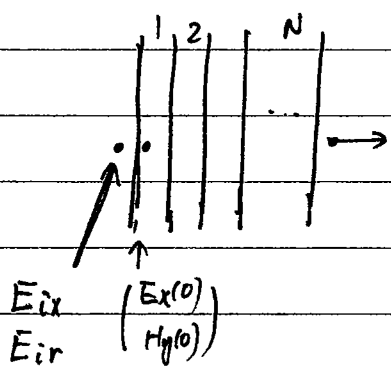
$$\begin{pmatrix} E_x(0) \\ H_y(0) \end{pmatrix} = M \begin{pmatrix} E_x(d) \\ H_y(d) \end{pmatrix}$$

$$M = \begin{pmatrix} \cos\varphi_2 & -ip_2 \sin\varphi_2 \\ -\frac{i}{p_2}\sin\varphi_2 & \cos\varphi_2 \end{pmatrix}$$

many layers (E, H) continuous.

$$M = M_1 M_2 \cdots M_n.$$

surface impedance
$$p_2 = \begin{cases} \dfrac{\cos\theta_2}{n_2/\mu c_0} & TM \\ -\dfrac{n_2\cos\theta_2}{\mu c_0} & TE. \end{cases}$$



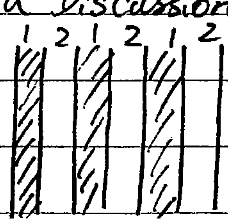
$$\begin{pmatrix} E_x(0) \\ H_y(0) \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ \frac{1}{p_1} & -\frac{1}{p_1} \end{pmatrix} \begin{pmatrix} E_{ix} \\ E_{rx} \end{pmatrix} = A \begin{pmatrix} E_{ix} \\ E_{rx} \end{pmatrix}$$

$$\begin{pmatrix} E_x(d) \\ H_y(d) \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{1}{p_3} \end{pmatrix} E_{tx} = B\, E_{tx}$$

$$\Rightarrow \quad A \begin{pmatrix} E_{ix} \\ E_{rx} \end{pmatrix} = M\, B\, E_{tx}$$

$$\begin{pmatrix} E_{ix} \\ E_{rx} \end{pmatrix} = A^{-1} M B\, E_{tx}.$$

Phenomena Discussion:



Bragg reflector.