2.58 HW4 Solutions

Lu Hu

Prob 1.

(a) (b) According to Drude model, the real and imaginary parts of the dielectric function are :

$$\varepsilon_r' = n^2 - k^2 = 1 - \frac{\omega_p^2}{\omega^2 + \gamma^2}$$

$$\varepsilon_r'' = 2nk = \frac{\gamma \omega_p^2}{\omega(\omega^2 + \gamma^2)}$$

We need to find ω_p and γ that minimize

$$\delta = \sum_i \left\{ [\varepsilon_r'(\omega_i) - \varepsilon_{rd}'(\omega_i)]^2 + [\varepsilon_r''(\omega_i) - \varepsilon_{rd}''(\omega_i)]^2 \right\}$$

where ε_{rd}' and ε_{rd}'' are the experimental data. This nonlinear least square fitting problem can be solved with MATLAB, for example, using "lsqnonlin". Curve fitting yields

$$\omega_p \approx 114.9 \times 10^{14} \text{ rad/s}$$

$$\gamma \approx 0.91 \times 10^{14} \text{ rad/s}$$

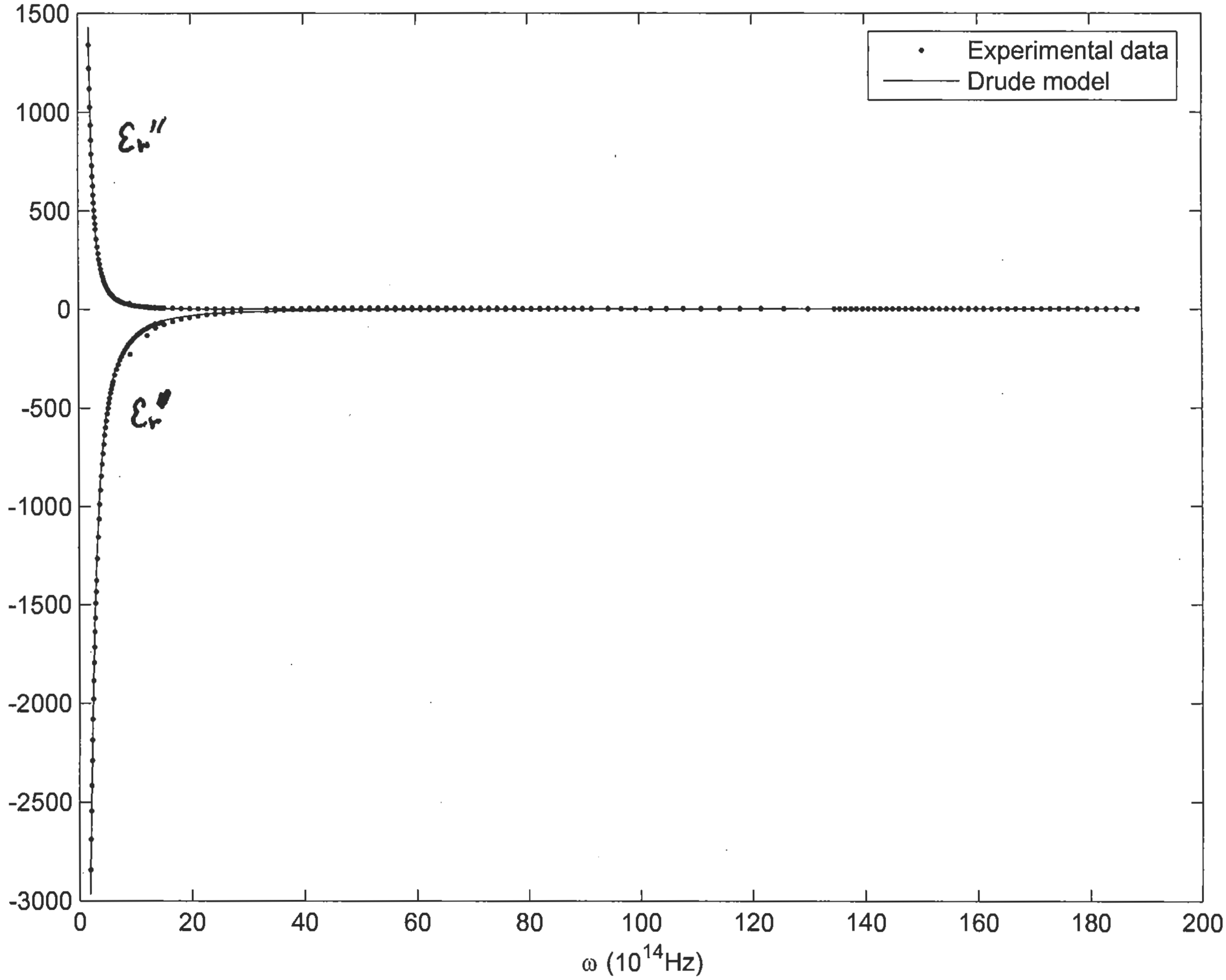
Note: you may obtain different values for ω_p and γ because of different minimization scheme. As long as ω_p and γ are in reasonable range, the fitting is acceptable. In addition, you could try "modified Drude model" which might give you better results.

(c)

$$\sigma_{e_{dc}} = \frac{\omega_p^2 \varepsilon_0}{\gamma} = 12.8 \times 10^6 \text{ /m.ohm}$$

The value in literature is 45.2×10^6 /m. ohm. The electric conductivity we obtain is in the right range.

(d) plots (next page)



1500
1000
500
0
-500
-1000
-1500
-2000
-2500
-3000
0
20
40
60
80
100
120
140
160
180
200
ω (10^14Hz)
Experimental data
Drude model
ε_r''
ε_r'

Prob 2. (a) Since the gold nanoparticles are very dilute, multiscattering can be neglected. Let's assume that the optimal wavelength scatisfies $x \ll 1$ so that the Rayleigh scattering model can be used.

$$Q_a \simeq Q_e = 4 \, Im\left(\frac{m^2-1}{m^2+1}\right) x = 4 \, Im\left(\frac{m^2-1}{m^2+1}\right) \cdot \frac{\omega \cdot n_w}{C_0} r$$

where n_w is the refractive index of water.

The energy absorbed by a gold nanoparticles is given by

$$\dot{g}_a = I_i \cdot Q_a \cdot A_c = \frac{4\pi r^3 \cdot I_i n_w \omega}{C_0} Im\left(\frac{m^2-1}{m^2+1}\right)$$

where $m^2 = \varepsilon / \varepsilon_w$ (w - water)

$$\frac{m^2-1}{m^2+2} = \frac{\varepsilon_r - \varepsilon_w}{\varepsilon_r + 2\varepsilon_w} = \frac{(\varepsilon_r' - \varepsilon_w) + i\varepsilon_r''}{(\varepsilon_r' + 2\varepsilon_w) + i\varepsilon_r''}$$

$$= \frac{(\text{Real}) + i[\varepsilon_r''(\varepsilon_r' + 2\varepsilon_w) - \varepsilon_r''(\varepsilon_r' - \varepsilon_w)]}{(\varepsilon_r' + 2\varepsilon_w)^2 + \varepsilon_r''^2} = \frac{\text{Real} + i \cdot 3\varepsilon_r'' \cdot \varepsilon_w}{(\varepsilon_r' + 2\varepsilon_w)^2 + \varepsilon_r''^2}$$

$$\Rightarrow \dot{g}_a = \frac{4\pi r^3 I_i n_w \omega}{C_0} \cdot \frac{3\varepsilon_r'' \cdot \varepsilon_w}{(\varepsilon_r' + 2\varepsilon_w)^2 + \varepsilon_r''^2}$$

We assume the particle loses heat by conduction only:

$$\dot{g}_c = 4\pi k r \cdot \Delta T$$

Energy balance gives

$$\Delta T = \frac{3 r^2 I_i n_w \omega \varepsilon_r'' \varepsilon_w}{k C_0[(\varepsilon_r' + 2\varepsilon_w)^2 + \varepsilon_r''^2]}$$

Recall that

$$\varepsilon_r' = 1 - \frac{\omega_p^2}{\omega^2 + \gamma^2}$$

$$\varepsilon_r'' = \frac{\gamma \omega_p^2}{\omega(\omega^2 + \gamma^2)}$$

$$\Rightarrow \Delta T = \frac{3r^2 I_i n_w \omega}{k c_0} \cdot \frac{\frac{\gamma \omega_p^2}{\omega(\omega^2+\gamma^2)} \cdot \varepsilon_w}{\left(1-\frac{\omega_p^2}{\omega^2+\gamma^2}+2\varepsilon_w\right)^2+\left(\frac{\gamma \omega_p^2}{\omega(\omega^2+\gamma^2)}\right)^2}$$

$$= \frac{3r^2 I_i n_w}{k c_0} \cdot \frac{\gamma \omega_p^2 \varepsilon_w}{(1+2\varepsilon_w)^2(\omega^2+\gamma^2) - 2\omega_p^2(1+2\varepsilon_w) + \frac{\omega_p^4}{\omega^2}}$$

(b)

Now we need to minimize the denominator:

$$f(\omega) = (\omega^2+\gamma^2)(2\varepsilon_w+1)^2 - 2(2\varepsilon_w+1)\omega_p^2 + \frac{\omega_p^4}{\omega^2}$$

$$\frac{df}{d\omega} = 2\omega(2\varepsilon_w+1)^2 - \frac{2\omega_p^4}{\omega^3} = 0 \Rightarrow \omega = \frac{\omega_p}{\sqrt{2\varepsilon_w+1}}$$

$$\frac{d^2f}{d\omega^2} = 2(2\varepsilon_w+1)^2 + \frac{6\omega_p^4}{\omega^4} > 0 \Rightarrow$$ It is indeed a minimum of $f(\omega)$, i.e. maximum of ΔT.

$$\lambda_{max} = \frac{2\pi c_0}{\omega_{max}} = \frac{2\pi c_0\sqrt{2\varepsilon_w+1}}{\omega_p} = \frac{2\pi \times 3\times 10^8 \times \sqrt{2\times 1.77+1}}{114.9\times 10^{14}} = 0.350\,\mu m$$

* λ_{max} for gold nanoparticles in vaccum should be around 500 nm. If we use the above formular and ω_p obtained from prob1, $\lambda_{max} = 0.$ μm We can use modified Drude model to improve the fitting. Neverthe We will use the standard Drude model for the following calculations.

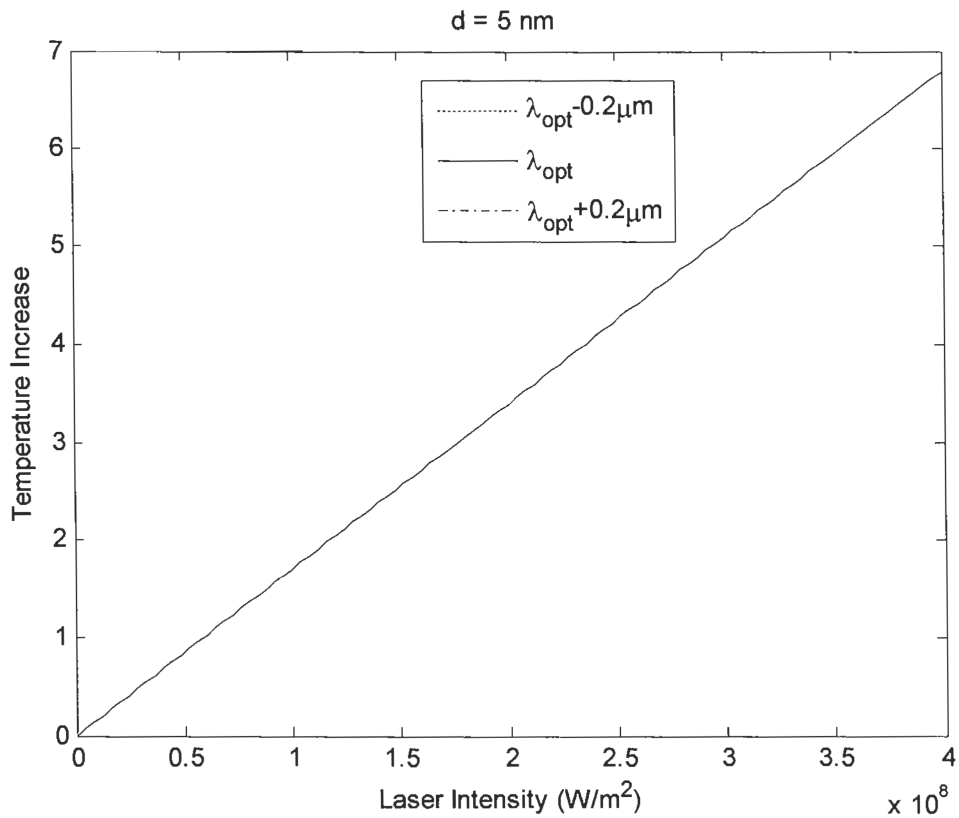
(C) Assume the laser intensity is uniform.

At the optimal wavelength

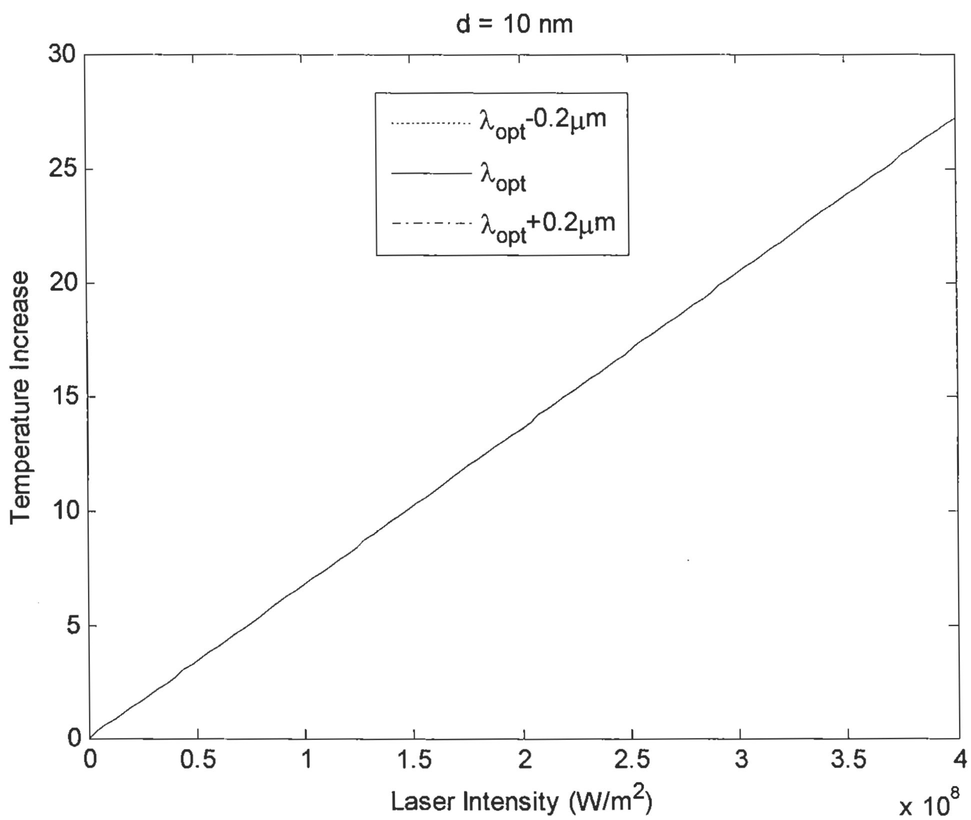
$$\varepsilon_r' = 1 - \frac{\omega_p^2}{\frac{\omega_p^2}{2\varepsilon_w+1}+\gamma^2} = -3.54$$

$$\varepsilon_r'' = \frac{\gamma \omega_p^2}{\frac{\omega_p}{\sqrt{2\varepsilon_w+1}}\left(\frac{\omega_p^2}{2\varepsilon_w+1}+\gamma^2\right)} = 0.0766$$

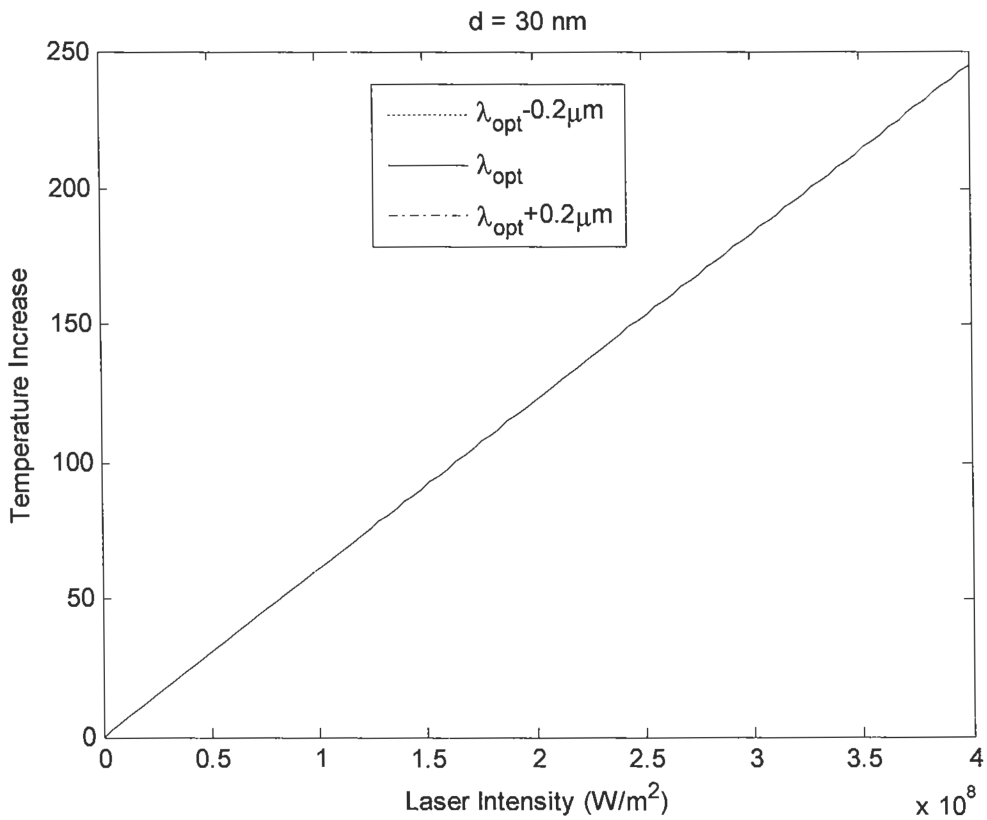
$\Rightarrow$



d = 5 nm
Temperature Increase
Laser Intensity (W/m^2)
x 10^8
λ_{opt}-0.2μm
λ_{opt}
λ_{opt}+0.2μm
0
1
2
3
4
5
6
7
0
0.5
1
1.5
2
2.5
3
3.5
4



d = 10 nm
Temperature Increase
Laser Intensity (W/m^2)
x 10^8
λ_{opt}-0.2μm
λ_{opt}
λ_{opt}+0.2μm
0
5
10
15
20
25
30
0
0.5
1
1.5
2
2.5
3
3.5
4



d = 30 nm
λ_{opt}-0.2μm
λ_{opt}
λ_{opt}+0.2μm
Temperature Increase
0
50
100
150
200
250
0
0.5
1
1.5
2
2.5
3
3.5
4
Laser Intensity (W/m^2)
x 10^8

At the optimal wavelength:

$$\Delta T = \frac{3r^2 I_i n_w \omega_p^2}{k c_0 \gamma} \cdot \frac{\varepsilon_w}{(2\varepsilon_w+1)^2}$$

$$\Rightarrow I_i = \Delta T \cdot \frac{k c_0 \gamma}{3r^2 n_w \omega_p^2} \cdot \frac{(2\varepsilon_w+1)^2}{\varepsilon_w} = 2.95\times10^8\ W/m^2$$

$$P_i = I_i \cdot \pi a^2 = 2.95\times10^8 \times \pi \times (2.5\times10^{-6})^2 = 5.8\ mW$$