Prob 1.8

Total power emitted from the bulb:

$$Q_{tot} = A_b \cdot \sigma T^4 = 100\ W \quad \cdots (1)$$

The visible part:

$$Q_v = A_b \int_{0.4\mu m}^{0.7\mu m} E_{b\lambda}\, d\lambda \quad \cdots (2)$$

Where $Q_v = (4\pi R^2)\cdot q_v$ and $q_v = 42.6\times10^{-3}\ W/m^2$ --- (3)

R is the distance from the bulb to the floor

Equations (1) – (3) give

$$\frac{Q_v}{Q_{tot}} = \frac{4\pi R^2 q_v}{Q_{tot}} = \frac{1}{\sigma T^4}\int_{0.4\mu m}^{0.7\mu m} E_{b\lambda}\, d\lambda$$

$$= \frac{1}{\sigma}\int_{0.4\mu m}^{0.7\mu m} \frac{C_1}{(\lambda T)^5\left[e^{C_2/\lambda T}-1\right]}\, d(\lambda T)$$

$\Rightarrow T \approx 2500.8\ K$ (Can be solved numerically or by trial and error using the table in Appendix C)

The efficiency of the bulb:

$$\eta = \frac{Q_v}{Q_{tot}} = \frac{4\pi R^2 q_v}{100} \approx 3.35\%$$

Prob. 1.11

The angle between the sun light and a normal to the window is:

$$\theta = \cos^{-1}(\cos 30^\circ \cos 45^\circ)$$

The incident flux density is then

$$q_{in} = q_s \cos\theta = 612.4\ W/m^2$$

Total hemispherical transmissivity:

$$\tau = \frac{\int_0^\infty \tau_\lambda q_{s,\lambda}\, d\lambda}{\int_0^\infty q_{s,\lambda}\, d\lambda} = \frac{\int_0^\infty \tau_\lambda E_{b\lambda}(T_{sun})\, d\lambda}{\sigma T_{sun}^4}$$, where $T_{sun} = 5777\ K$

(1)

(a) For the plain glass

$$\tau = 0.90\,[f(T_{sun}\times 2.7\,\mu m) - f(T_{sun}\times 0.35\,\mu m)]$$
$$= 0.90\,[0.97195 - 0.07017) = 0.81160$$

$$q_{trans} = \tau q_{in} = 0.8116 \times 612.4 = 497.0\ W/m^2$$
$$q_{abs} = (1-\rho-\tau)\,q_{in} = 66.4\ W/m^2$$
$$q_{ref} = \rho q_{in} = 49.0\ W/m^2$$

(b) For the tinted glass

$$\tau = 0.90\,[f(T_{sun}\times 1.4\,\mu m) - f(T_{sun}\times 0.5\,\mu m)]$$
$$= 0.90\,[0.85970 - 0.24794) = 0.55058$$

$$\Rightarrow \quad q_{trans} = \tau q_{in} = 337.2\ W/m^2$$
$$q_{ref} = \rho q_{in} = 49.0\ W/m^2$$
$$q_{abs} = (1-\rho-\tau)\,q_{in} = \cancel{188.8\ W/m^2}\ 226.2\ W/m^2$$

(c) Transmitted visible light through the plain glass:

$$q_{trans,\,plain} = \tau_{visible,\,plain}\, q_{in} = q_{in}\times 0.9\,[f(T_{sun}\times 0.7\,\mu m) - f(T_{sun}\times 0.4\,\mu m)]$$

Transmitted visible light through the tinted glass:

$$q_{trans,\,tinted} = \tau_{visible,\,tinted}\, q_{in} = q_{in}\times 0.9\,[f(T_{sun}\times 0.7\,\mu m) - f(T_{sun}\times 0.5\,\mu m)]$$

$$\Rightarrow \frac{q_{trans,\,plain} - q_{trans,\,tinted}}{q_{trans,\,plain}} = \frac{f(T_{sun}\times 0.5\,\mu m) - f(T_{sun}\times 0.4\,\mu m)}{f(T_{sun}\times 0.7\,\mu m) - f(T_{sun}\times 0.5\,\mu m)}$$

$$= 34.3\%$$

However, according to Fig. 11, human eyes are not very sensitive to wavelength below 0.5 μm. The effective reduction by the tinted glass is thus much less.

Prob. 3.4

(a) The total hemispherical emittance given by the Planck's law:

$$\varepsilon(T) = \frac{0.5}{\sigma T^4}\left[\int_0^{\lambda_c} E_{b\lambda} d\lambda + \int_{\lambda_c}^{\infty} \frac{\lambda_c}{\lambda} E_{b\lambda} d\lambda\right]$$

$$= 0.5\cancel{f(\lambda_c T)}^{\sim 0} + \frac{0.5}{\sigma T^4}\int_{\lambda_c}^{\infty} \frac{\lambda_c C_1}{\lambda^6 (e^{C_2/\lambda T}-1)} d\lambda$$

$$= \frac{0.5\lambda_c C_1 T}{\sigma C_2^5}\int_0^{\frac{C_2}{\lambda_c T}} \frac{x^4}{e^x-1} dx$$

$\Rightarrow$ $\varepsilon(300k) = 0.01998$, $\varepsilon(1000k) = 0.06659$

(b) By Wien's law:

$$\varepsilon(T) = \frac{\int_0^{\infty} \varepsilon_\lambda \frac{C_1}{\lambda^5} e^{-C_2/\lambda T} d\lambda}{\int_0^{\infty} \frac{C_1}{\lambda^5} e^{-C_2/\lambda T} d\lambda} \approx \frac{0.5\lambda_c \int_0^{\frac{C_2}{\lambda_c T}} \frac{T^6 x^5}{C_2^5} e^{-x} dx}{\int_0^{\infty} \frac{T^5 x^3}{C_2^4} e^{-x} dx}$$

Note $\frac{C_2}{\lambda_c T} >> 1$ for both temperatures.

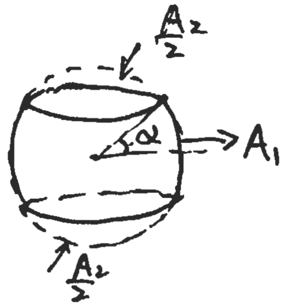
$$\Rightarrow \varepsilon(T) = \frac{0.5 T\lambda_c}{C_2} \cdot \frac{\int_0^{\infty} x^4 e^{-x} dx}{\int_0^{\infty} x^3 e^{-x} dx} = \frac{0.5 T\lambda_c}{C_2} \cdot \frac{4!}{3!}$$

$$= 6.9502\times 10^{-5}\, T/k$$

$\varepsilon(300k) = 0.02085$, error $= 4.4\%$

$\varepsilon(1000k) = 0.06950$, error $= 4.4\%$

Prob 4.16



$$F_{1-2} = \frac{A_2}{A_s} \Rightarrow F_{1-1} = 1 - \frac{A_2}{A_s} = \frac{A_1}{A_s}$$

where $A_1 = \int_{\frac{\pi}{2}-\alpha}^{\frac{\pi}{2}+\alpha}\int_0^{2\pi} R^2 \sin\theta \, d\theta \, d\varphi = 4\pi R^2 \sin\alpha$

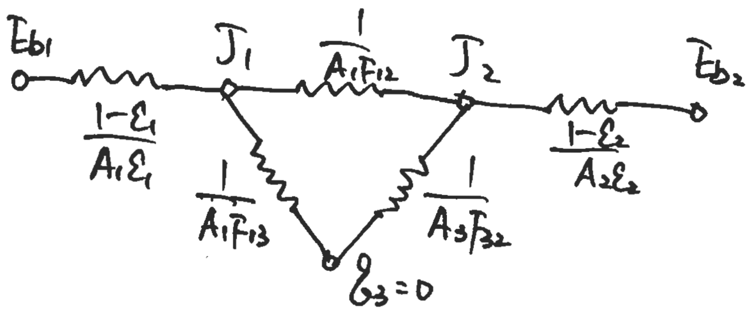
$$F_{1-1} = \frac{4\pi R^2 \sin\alpha}{4\pi R^2} = \sin\alpha$$

Prob 5.13

From Appendix D, configuration 51:

$$F_{1-3} = F_{1-2} = \frac{1}{\pi}\left(\cos^{-1}\frac{1}{2} + 2 - \sqrt{4-1}\right) \approx 0.4186$$

$$F_{32} = 1 - F_{31} = 1 - \frac{A_1}{A_3}F_{13} = 1 - \frac{\pi d}{s}F_{13} = 0.3425$$



$$E_{b1} = E_{b2} + Q_2\left[\frac{1-\varepsilon_1}{A_1\varepsilon_1} + \frac{1}{A_1F_{12} + \left(\frac{1}{A_1F_{13}} + \frac{1}{A_3F_{32}}\right)^{-1}} + \frac{1-\varepsilon_2}{A_2\varepsilon_2}\right]$$

$$= \sigma T_2^4 + q_2\left[\frac{A_2}{A_1}\cdot\frac{1-\varepsilon_1}{\varepsilon_1} + \frac{1}{\frac{A_1}{A_2}F_{12} + \left(\frac{A_2}{A_1}\frac{1}{F_{13}} + \frac{A_2}{A_3}\cdot\frac{1}{F_{32}}\right)^{-1}} + \frac{1-\varepsilon_2}{\varepsilon_2}\right]$$

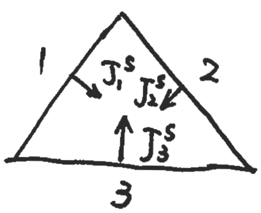
$$= 7.495 \times 10^5 \ W/m^2$$

$$T_1 = \left(\frac{E_{b1}}{\sigma}\right)^{\frac{1}{4}} = 1906.8 K$$

Prob 5.18

The incident Solar radiation on A_3 is

$$H_{o3} = (1-\rho_1)\, q_{sun} \cos 30^\circ = 779.4\ W/m^2$$



Note that part of the incident solar radiation will leave the green house through A_1.
We define the "solar radiosity" as J^s.

$(A_1 = A_2 = A_3 = A = 1)$

$$J_3^s = \rho_3 (H_{o3} + J_1^s F_{13} + J_2^s F_{23})$$

$$J_1^s = \rho_1 (J_3^s F_{31} + J_2^s F_{21})$$

$$J_2^s = \rho_2 (J_1^s F_{12} + J_3^s F_{32})$$

where $\rho_1 = 0.1$, $\rho_2 = 1-\varepsilon_2 = 0.8$, $\rho_3 = 1 - \varepsilon_3 = 0.2$

$$F_{12} = F_{21} = F_{31} = 1 - \sin\frac{60^\circ}{2} = 0.5$$

$\Rightarrow J_1^s = 11.7\ W/m^2$, $J_2^s = 69.9\ W/m^2$, $J_3^s = 163.2\ W/m^2$

Solar energy leaving the greenhouse

$$Q_{leave} = (1-\rho_1)(J_3 F_{31} A + J_2 F_{21} A) = 104.9\ W$$

Neglecting heat loss from surfaces 1 and 2 to the outside, we have

$$Q_{abs} = |q_3| A = 779.4 - 104.9 = 674.5\ W$$

$$\Rightarrow T_3 = \frac{|q_3|}{U} + T_\infty = \frac{674.5}{19.5} + 280 = 314.6\ K$$

The radiosity leaving surface 3 consists of 2 parts: the solar part and the IR part.

$$J_3 = J_3^s + J_3^i$$

According to Eq. 5.27 (Surface 3 is gray so that 5.27 is valid)

$$J_3 = E_{b3} - \left(\frac{1}{\varepsilon_3} - 1\right) q_3 = 724.04\ W$$

$J_3^i = 724.04 - J_3^s = 560.8 \ W/m^2$

For surface 2, $q_2 = 0$, Eq. (5.26) gives

$$J_2 = E_{b2} \quad \text{---} \ (1)$$

For surface 1, $q_1 = 0$, energy balance gives

$$0 = \varepsilon_1 E_{b1} - \int \alpha_\lambda H_\lambda d\lambda = J_1^i - H_1^i = \varepsilon_1 E_{b1} - \varepsilon_1 H_1^i$$

$$\Rightarrow \quad J_1^i = E_{b1} \quad \text{---} \ (2)$$

For surfaces 1 & 2, $\begin{cases} q_2 = J_2 - H_2 \\ q_1 = J_1^i - H_1^i \end{cases}$

$$\Rightarrow \begin{cases} 0 = J_2 - (J_3 F_{32} + J_1 F_{12}) \\ 0 = J_1^i - (J_3^i F_{31} + J_2^i F_{21}) \end{cases}$$

$$\Rightarrow \begin{cases} J_2^i + J_2^s = J_3 F_{32} + (J_1^i + J_1^s) F_{12} \\ J_1^i = J_3^i F_{31} + J_2^i F_{21} \end{cases}$$

Plug in numbers:

$$\begin{cases} J_2^i + 69.9 = 724.04 \times 0.5 + (J_1^i + 11.7) \times 0.5 \\ J_1^i = 560.8 \times 0.5 + 0.5 J_2^i \end{cases}$$

$$\Rightarrow \quad J_1^i = 572.5 \ W/m^2, \quad J_2^i = 584.2 \ W/m^2$$

$$\Rightarrow T_1 = \left(\frac{E_{b1}}{\sigma}\right)^{1/4} = \left(\frac{J_1^i}{\sigma}\right)^{1/4} = 317.0 \ K$$

$$T_2 = \left(\frac{E_{b2}}{\sigma}\right)^{1/4} = \left(\frac{J_2}{\sigma}\right)^{1/4} = \left(\frac{584.2 + 69.9}{\sigma}\right)^{1/4} = 327.7 \ K$$