MASSACHUSETTS INSTITUTE OF TECHNOLOGY

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Quantum Computation

Problem 1. For the state $|\psi\rangle = \frac{1}{2^{n/2}} \sum_{x=0}^{2^n-1} (-1)^{f(x)} |x\rangle_1 |g(x)\rangle_2$, where $g(x)$ is a 1-1 function, find the partial trace $\rho_1 \equiv tr_2(|\psi\rangle\langle\psi|)$ and calculate ${}^{\otimes n}\langle + | \rho_1 | + \rangle^{\otimes n}$.

Problem 2. Find $H^{\otimes n} R_\alpha H^{\otimes n}$ and $H^{\otimes n} T_\alpha H^{\otimes n}$ in simpler terms, where

$$R_\alpha = \sum_{x=0}^{2^n-1} (-1)^{x \bullet \alpha} |x\rangle\langle x|$$

and

$$T_\alpha = \sum_{x=0}^{2^n-1} |x \oplus \alpha\rangle\langle x|.$$

Problem 3. Find $U_p R_p U_p^\dagger$ and $U_p T_p U_p^\dagger$ in simpler terms, where

$$R_p = \sum_{x=0}^{p-1} \exp(2\pi x i / p) |x\rangle\langle x|$$

$$T_p = \sum_{x=0}^{p-1} |x + 1 \bmod p\rangle\langle x|$$

$$U_p = \frac{1}{\sqrt{p}} \sum_{x=0}^{p-1} \sum_{y=0}^{p-1} \exp(2\pi i x y / p) |y\rangle\langle x|$$

and p is a prime number.

Problem 4. Show that $U_2 \otimes U_3 = P U_6 P^{-1}$ where U_p is defined in Problem 3, and P is a permutation matrix (a matrix with only one nonzero element 1 in each row and column).