Problem set 5

Suggested reading: Baker and Gollub, Section 2.2 and Chapter 3.

Use `Matlab` to do the numerical computation in this problem set. Copy the files `drvpend.m` and `drvpend.c` to your working directory. You have to compile drvpend.c before using Matlab, so please read the supplement to this problem set for instructions.

1. The nonlinear parametric pendulum is described by

$$\frac{d^2\theta}{dt^2} + 2\gamma\frac{d\theta}{dt} + \omega_0^2[1 + h\cos 2(\omega_0 + \epsilon)t]\sin\theta = 0$$

For this problem choose $\omega_0 = 1$. Unless otherwise specified, use $\gamma = 0.1$ and $\epsilon = 0$.

a) For the unforced, undamped pendulum ($h = \gamma = 0$) find the time series and power spectrum of $\theta(t)$ for the initial conditions

i) $\dot\theta(0) = 0$, $\theta(0) = 0.01$.

ii) $\dot\theta(0) = 0$, $\theta(0) = 3.0$.

What are the differences between the two spectra? What is the source of these differences?

b) In class we showed that, when $\epsilon \simeq 0$, the rest state of the pendulum is unstable when $\gamma^2 < [(h\omega_0/2)^2 - \epsilon^2]/4$. Choosing either the resonant ($\epsilon = 0$) or the near-resonance case, verify your prediction of the critical value of h necessary for sustained oscillations of the pendulum. Calculate the power spectrum and Poincaré section for these oscillations. When constructing your Poincaré section, you may find it useful to use a natural frequency of the system, i.e. ω_0. Describe how the Poincaré section relates to the time series and the peaks in the power spectrum.

c) For $h = 1.5$, using the time series, power spectrum of $\dot\theta(t)$ and Poincaré section, explain why the period of $\dot\theta(t)$ is now π instead of 2π.

d) Compare the time series of $\dot\theta(t)$ for $h = 1.8$ to that of $h = 1.5$ in (c). What is the period of $\dot\theta(t)$ now?

e) Using either the time series of $\dot\theta(t)$, the power spectrum of $\dot\theta(t)$, or the Poincaré section, find the period of the pendulum for $h = 2.00$, 2.05 and 2.062 (remember to let the initial transients die away).

f) For $h = 2.2$ the motion of the pendulum is aperiodic. Generate $\theta(t)$, the power spectrum of $\dot\theta(t)$ and the Poincaré section for $h = 2.2$. What characteristics of these plots indicate aperiodicity?

g) The motion of the pendulum for $h = 2.2$ is on a “strange attractor”. By varying the initial conditions slightly, show that long term prediction of the pendulum’s motion is sensitive to the initial conditions.