## Massachusetts Institute of Technology
Biological Engineering Division
Department of Mechanical Engineering
Department of Electrical Engineering and Computer Science

## 2.797/20.310/3.0536.024, Fall 2006
## MOLECULAR, CELLULAR, & TISSUE BIOMECHANICS

Problem Set #7 Issued: 11/10/06
Due: 11/16/06

### Problem 1

A sample of soft tissue is cut into a cylindrical specimen and tested under constant uniaxial strain ($\varepsilon_{11}$ = 10%) applied at $t$ = 0 and held constant, as shown in the figure. The measured stress history is also shown.

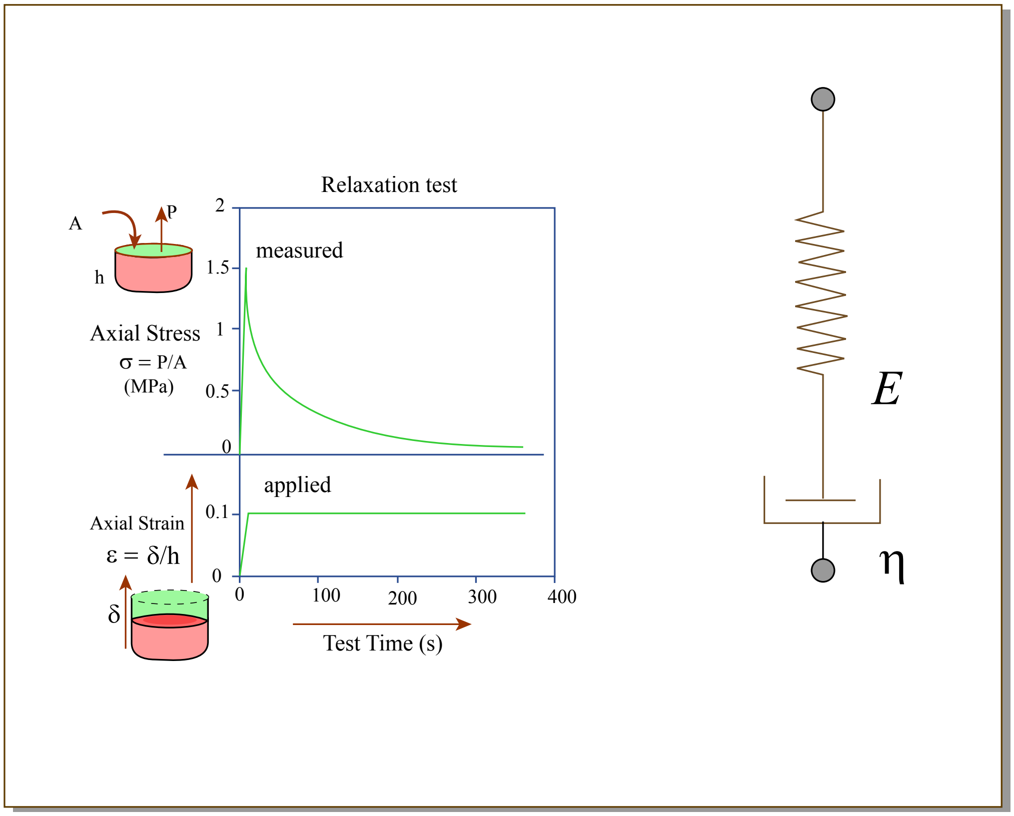

Figure by MIT OCW.

a) Assuming that the material behaves as a Maxwell model, obtain numerical estimates for the Maxwell material properties, $E$ and $\eta$, based on these test data.

b) Now, use the Maxwell model to predict the response of the material in terms of the time-dependent strain [$\varepsilon(t)$], in response to a ramp in stress given by:

$$\sigma(t) = \alpha \cdot t$$

where $\alpha = 1$ MPa for the time $0 \le t \le 100s$. Use the values you obtained above for $E$ and $\eta$ to plot your result in numerical form for the same time interval. Do this in two ways. First, by using the general expression relating $\varepsilon(t)$ to the integral containing the creep compliance:

$$\varepsilon(t) = \int_0^t J(t-\tau)\frac{d\sigma}{d\tau}d\tau$$

and second, by solving the governing differential equation relating stress and strain:

$$\dot{\varepsilon} = \dot{\sigma}/_E + \sigma/_\eta$$

with the appropriate initial conditions.

## Problem 2

We discussed in class the experiments by Desprat, et al., in which a cell is tethered to two surfaces, and deformed under a constant stress of $\sigma_0$ (see figure). Note that the authors fit the experimental data to two different models, one comprised of springs and dashpots, and the other described simply in terms of the creep compliance:

$$J(t) \equiv \frac{\varepsilon(t)}{\sigma_0} = kt^{\alpha}$$

where $k$ and $\alpha$ are parameters determined from the fit to the experimental data.

a) Using the data shown in the plot below (open circles), determine the values of $k$ and $\alpha$. The initial level of stress, $\sigma_0$, is 100 Pa.

b) As a practical matter, it is difficult to apply the stress as a true step function. If the stress is applied as a linear ramp from 0-100 Pa over a period of 100 ms, then held constant, compute the corresponding strain. Plot this strain as a function of time on a linear scale for the period 0 < t < 500 ms.

c) Consider now the spring and dashpot model representation shown below. Recognizing that this model is simply the sum of a Kelvin (SLS) model and a dashpot in series, and given the relationship between the stress and strain for the Kelvin model from class notes, obtain the governing differential equation for this system. (Note: by using the Kelvin model result, in combination with the constitutive law for a single dashpot, you can reduce the algebra considerably!)

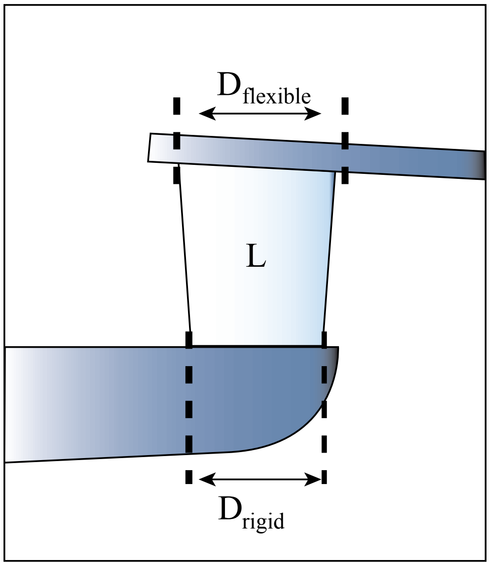

Figure by MIT OCW.

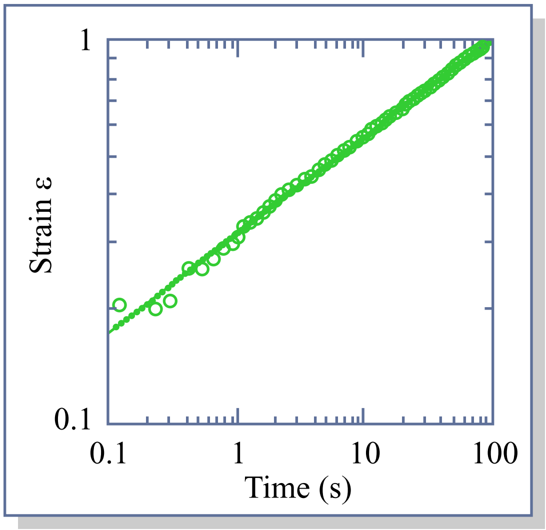

Figure by MIT OCW.

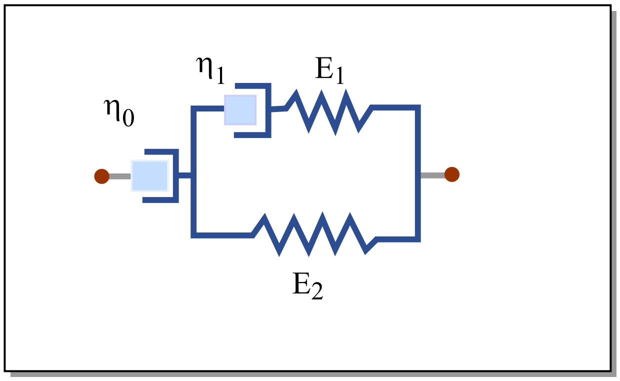

Figure by MIT OCW.

d) Using physical reasoning, explain the behavior of the dashpot model in terms of *G'* and *G"* in the limits of high and low frequency. In each limit, give an estimate for *G'* and *G"* in terms of the parameters of the model defined in the figure.