Some useful definitions

- **System:** That which is to be described, analyzed & controlled—anything of interest which is to be described in detail. Often defined as a collection of objects enclosed by a boundary, but this is not essential and the boundary may be conceptual rather than tangible.
- **Environment:** All that is external to the system[1]. Everything else of interest, but which will not be described in detail. Commonly conceived as external to the system, but again, this is not essential.
- **Open, closed:** The behavior of an open system may depend upon its environment; i.e., the two interact. A closed system does not interact with its environment.
- **System Variable:** A quantity, used to describe the system, which may change with time (or space).
- **System Input:** A quantity that is prescribed or imposed on the system by the environment; i.e. an independent variable.
- **System Output:** *Any* system variable of interest.
- **State Determined Systems (SDS):** A class of systems fully determined by a finite set of *state variables* $(x_1, x_2, \ldots x_n)$.
- **State:** A minimal, complete and independent set of state variables $(x_1, x_2, \ldots x_n)$ that uniquely describe the system.
- **State Equations:** To describe a state-determined system's behavior uniquely for all $t>t_0$ it is sufficient to have:
  (i) Values of a finite set of variables $(x_1, x_2, \ldots x_n)$ at t_0,
  (ii) Values of a finite set system inputs $(u_1, u_2, \ldots u_r)$ for all $t>t_0$, and
  (iii) A set of state equations:
  $$\begin{aligned} dx_1/dt &= f_1(x_1, x_2, \ldots x_n, u_1, u_2, \ldots u_r, t) \\ dx_2/dt &= f_2(x_1, x_2, \ldots x_n, u_1, u_2, \ldots u_r, t) \\ &\vdots \\ dx_n/dt &= f_n(x_1, x_2, \ldots x_n, u_1, u_2, \ldots u_r, t) \end{aligned}$$
- **Output equations:** Any output variables $(y_1, y_2, \ldots y_m)$ of a state-determined system may be expressed as functions of its state and input variables:
  $$\begin{aligned} y_1 &= g_1(x_1, x_2, \ldots x_n, u_1, u_2, \ldots u_r, t) \\ y_2 &= g_2(x_1, x_2, \ldots x_n, u_1, u_2, \ldots u_r, t) \\ &\vdots \\ y_m &= g_m(x_1, x_2, \ldots x_n, u_1, u_2, \ldots u_r, t) \end{aligned}$$

[1] It should be clear that the distinction between "system" and "environment" is not a property of the real world, but a matter of descriptive convenience. Any given object may be described as part of a system in one situation, and part of the environment in another.

Vector notation

A more compact notation is as follows.

- **State Space:** An abstract n-dimensional space defined by the state variables.
- **State Vector:** A point in state space defined by a complete set of state variables $\mathbf{x} = (x_1, x_2, \ldots x_n)^t$.
- **Input Space:** An abstract r-dimensional space defined by the input variables.
- **Input Vector:** A point in input space defined by a complete set of input variables $\mathbf{u} = (u_1, u_2, \ldots u_r)^t$.
- **Output Space:** An abstract m-dimensional space defined by the output variables.
- **Output Vector:** A point in output space defined by a complete set of output variables $\mathbf{y} = (y_1, y_2, \ldots y_m)^t$.
- **State Equations:** $d\mathbf{x}/dt = \mathbf{f}(\mathbf{x}, \mathbf{u}, t)$
- **Output Equations:** $\mathbf{y} = \mathbf{g}(\mathbf{x}, \mathbf{u}, t)$