MASSACHUSETTS INSTITUTE OF TECHNOLOGY
DEPARTMENT OF MECHANICAL ENGINEERING
CAMBRIDGE, MASSACHUSETTS 02139

2.002 MECHANICS AND MATERIALS II
SPRING, 2003

PRACTICE QUIZ II

Distributed: Wednesday, April 28, 2004

This quiz consists of four (4) questions. A brief summary of each question's content and associated points is given below:

1. (15 points) This is the credit for your (up to) two (2) pages of self-prepared notes. Please be sure to put your name on each sheet, and hand it in with the test booklet. You are already done with this one!
2. (15 points) A lab-based question.
3. (20 points) A set of 4 short, straightforward questions about plasticity.
4. (50 points) A longer problem on fatigue crack growth and fracture.

The last 2 pages of the quiz contain "useful" information. Please refer to these pages for equations, etc., as needed.

If you have any questions about the quiz, please ask for clarification.

Good luck!

Problem 1 (15 points)

Attach your self-prepared 2-sheet (4-page) notes/outline to the quiz booklet. Be sure your name is on each sheet.

Problem 2 (15 points)

(Lab Problem)

A strip of 2024-T4 aluminum alloy is $3\,mm$ thick and $30\,mm$ wide. Material properties for the alloy include $E = 70 \times 10^3\,MPa$, $\nu = 0.3$, and $\sigma_y = 300\,MPa$.

The strip is to be bent over a solid circular cylindrical die of radius "R". Answer the following questions:

- (**5 points**) What is the smallest value the radius "R" can have so that the strip of 2024-T4 aluminum **fully springs back to its original shape** after having been wrapped around the die?
- (**5 points**) The aluminum strip is bent around a die having outer radius $R = 30\,mm$. What value would an axially-mounted strain gage on the outer surface of the strip record when the strip is wrapped around the die?
- (**5 points**) Estimate the value that the strain gage above would register after unloading from being wrapped about the die of radius $R = 30\,mm$.

Show all work and state all assumptions.

Problem 3 (20 points) An elastic-plastic material has Young's modulus E and shear modulus G. Under uniaxial tension, plastic deformation begins when the axial stress reaches the value "σ_y", and the "shape" of the tensile stress versus tensile strain curve is well-approximated as bi-linear, with <u>constant</u> post-yield slope $d\sigma/d\epsilon = E_t$, as indicated in the figure. Assume that plasticity in this material is described by Mises yield condition along with isotropic strain hardening.

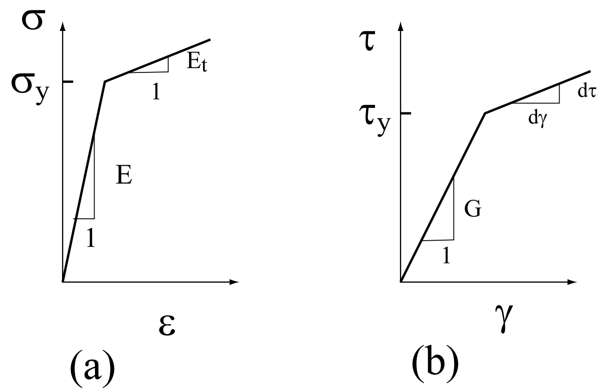

Figure 1: Idealized stress-strain curves. (a) (**Tension**) tensile yield strength: "σ_y"; post-yield slope: $d\sigma/d\epsilon = E_t$.(b) (**Shear**) shear yield strength: τ_y =?; post-yield slope: $d\tau/d\gamma$ =?.

1. (**10 points**) Describe this material's constitutive relation between deformation resistance (or "strength"), s, and equivalent tensile plastic strain, $\bar{\epsilon}_p$. In particular,
   (a) What is the initial value of s, evaluated at $\bar{\epsilon}_p = 0$?
   (b) What is the slope, h, of the function $s(\bar{\epsilon}_p)$? (Note: $h \equiv ds(\bar{\epsilon}_p)/d\bar{\epsilon}_p$.)

2. (**10 points**) A coupon of this material is subjected to a state of stress in pure shear, with stress components given, for some value of the shear stress, "τ," by

$$[\sigma_{ij}] = \begin{bmatrix} 0 & \tau & 0 \\ \tau & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

   (a) At what value of $\tau = \sigma_{12}$ will the material start to yield plastically? (The value of shear stress at yield can be denoted "τ_y".)
   (b) Derive an expression for the initial post-yield slope of the shear stress versus shear strain curve. That is, give an expression for the quantity

$$\frac{1}{2}\frac{d\sigma_{12}}{d\epsilon_{12}} = \frac{d\sigma_{12}}{2\,d\epsilon_{12}} = \frac{d\sigma_{12}}{d\gamma_{12}} \equiv \frac{d\tau}{d\gamma}.$$

   Here we have used the usual relation between engineering and tensorial shear strains, $2\epsilon_{12} = \gamma_{12} \equiv \gamma$, etc. State all assumptions.

Problem 4 (50 points)

The martensitic stainless steel 431 is heat-treated to a room-temperature yield strength of $\sigma_y = 700\,MPa$ and plane strain fracture toughness $K_{Ic} = 110\,MPa\sqrt{m}$. Fatigue crack propagation in 431 stainless steel has a power-law exponent of $m = 2$, and the growth rate per cycle is $da/dN = \Delta a_o = 10^{-4}\,mm/$cycle when the applied cyclic range of the stress intensity factor is $\Delta K_I = \Delta K_{Io} = 20 MPa\sqrt{m}$.

A component of this 431 stainless steel is to be cycled at room temperature between zero stress and a maximum tensile stress of $\sigma_{\max}$ (to-be determined). Loading will be applied for $25,000$ cycles. It is required that **any pre-existing edge cracks of size** $a_i \leq 3\,mm$ **must not grow beyond a size which would give a maximum** K_I**-value greater than** $0.75 \times K_{Ic}$, resulting in a minimum safety factor against fracture of 4/3.

- **(40 points)** Please choose the largest possible value for $\sigma_{\max}$ consistent with this constraint and any other constraint(s) you might deem appropriate. You may assume that the structure is sufficiently wide ($w/a \gg 1$) so that the configuration correction factor Q can be taken as constant, $Q = 1.12$.
- **(10 points)** Suppose that you really <u>did</u> have an initial crack of size $a_i = 3\,mm$ and, further, that you really <u>did</u> apply $25,000$ cycles of the zero/max stressing at the value of $\sigma_{\max}$ determined above. How many more cycles of $0/\sigma_{\max}$ loading would extend the crack to critical size? Does the proposed "safety factor" of 4/3 on fracture seem appropriate? Discuss.

Isotropic Linear Thermal-Elasticity
(Cartesian Coordinates)

Stress/Strain/Temperature-Change Relations:

$$\epsilon_{ij} = \alpha \Delta T\, \delta_{ij} + \frac{1}{E} \left[(1+\nu)\, \sigma_{ij} - \nu \left(\sum_{k=1}^{3} \sigma_{kk} \right) \delta_{ij} \right].$$

$$\sigma_{ij} = \frac{E}{(1+\nu)} \left[\epsilon_{ij} + \frac{\nu}{(1-2\nu)} \left(\sum_{m=1}^{3} \epsilon_{mm} \right) \delta_{ij} - \frac{(1+\nu)}{(1-2\nu)} \alpha \Delta T\, \delta_{ij} \right].$$

Strain-displacement Relations:

$$\epsilon_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right).$$

Equilibrium equations (with body force and acceleration):

$$\sum_{j=1}^{3} \frac{\partial \sigma_{ij}}{\partial x_j} + \rho b_i = \rho \frac{\partial^2 u_i}{\partial t^2}$$

Isotropic Elasto-Plasticity
(Cartesian Coordinates)

Mises Equivalent Tensile Stress Measure:

$$\begin{aligned} \bar{\sigma} &\equiv \sqrt{\frac{1}{2} \left[(\sigma_{11}-\sigma_{22})^2 + (\sigma_{22}-\sigma_{33})^2 + (\sigma_{33}-\sigma_{11})^2 \right] + 3 \left[\sigma_{12}^2 + \sigma_{23}^2 + \sigma_{31}^2 \right]} \\ &= \sqrt{\frac{3}{2} \sum_{i=1}^{3} \sum_{j=1}^{3} \sigma'_{ij} \sigma'_{ij}}. \end{aligned}$$

Yield Condition:

$$\bar{\sigma} \le s.$$

Stress Deviator Tensor:

$$\sigma'_{ij} = \sigma_{ij} - \frac{1}{3} \delta_{ij} \left(\sum_{k=1}^{3} \sigma_{kk} \right).$$

Elastic-Plastic Strain-Increment Decomposition:

$$d\epsilon_{ij} = d\epsilon_{ij}^{(e)} + d\epsilon_{ij}^{(p)}.$$

Plastic flow rule:

$$d\epsilon_{ij}^{(p)} = \frac{3}{2}\, d\bar{\epsilon}_p\, \frac{\sigma'_{ij}}{\bar{\sigma}}.$$

Tensile equivalent plastic strain increment:

$$d\bar{\epsilon}_p = \sqrt{\frac{2}{3}\sum_{i=1}^{3}\sum_{j=1}^{3} d\epsilon_{ij}^{(p)} d\epsilon_{ij}^{(p)}} \geq 0.$$

Strain Hardening:

$$ds = h\, d\bar{\epsilon}_p.$$

Power-law Fatigue Crack Growth under Constant-$\Delta\sigma$ and Constant-Q:

(a) **power-law exponent** $m > 2$**:**

Cycles to propagate between fixed crack lengths:

$$N_{a_i \to a_f} = \frac{a_i}{\Delta a_o}\left(\frac{\Delta K_{Io}}{Q\Delta\sigma\sqrt{\pi a_i}}\right)^m \frac{2}{(m-2)}\left[1 - \left(\frac{a_i}{a_f}\right)^{\frac{(m-2)}{2}}\right]; (m > 2).$$

Fatigue crack length vs. cycles:

$$a(N) = \frac{a_i}{\left(1 - \frac{N}{N_0}\right)^{\frac{2}{(m-2)}}},$$

where

$$N_0 \equiv \frac{a_i}{\Delta a_o}\left(\frac{\Delta K_{Io}}{Q\Delta\sigma\sqrt{\pi a_i}}\right)^m \frac{2}{(m-2)}.$$

(b) **power-law exponent** $m = 2$**:**

Cycles to propagate between fixed crack lengths:

$$N_{a_i \to a_f} = \frac{a_i}{\Delta a_o}\left(\frac{\Delta K_{Io}}{Q\Delta\sigma\sqrt{\pi a_i}}\right)^2 \left[\ln\left(\frac{a_f}{a_i}\right)\right]$$

Fatigue crack length vs. cycles:

$$a(N) = a_i \exp\left(N\left[Q\,\Delta\sigma\sqrt{\pi\Delta a_o}/\Delta K_{Io}\right]^2\right).$$