

## gear automation and design revised

Backlash\_lookup<sub>6</sub> := 0.015      Backlash\_lookup<sub>12</sub> := 0.009      Backlash\_lookup<sub>18</sub> := 0.007  
 Backlash\_lookup<sub>8</sub> := 0.010      Backlash\_lookup<sub>14</sub> := 0.009      Backlash\_lookup<sub>20</sub> := 0.006  
 Backlash\_lookup<sub>10</sub> := 0.010      Backlash\_lookup<sub>16</sub> := 0.008

Table page 68 Lynwander, Gear Drive Systems for backlash at various diametral values

numerical example ...

DP := 10      diametral\_pitch      middle of course range 2 .... 16 per Shigley Table 9.3

φ<sub>1</sub> := 20deg      pressure\_angle\_at\_pitch\_radius      addendum :=  $\frac{1}{DP}$   
 N<sub>P</sub> := 20      number\_of\_pinion\_teeth      dedendum :=  $\frac{1.25}{DP}$       Table 9.2 Sigley for pressure angles 20, 22.5 and 25  
 N<sub>G</sub> := 30      number\_of\_gear\_teeth

\*\*\*\*\* end of input \*\*\*\*\*

$C := \frac{N_P + N_G}{2 \cdot DP}$       C = 2.5      center\_distance       $R := \frac{N_G}{N_P}$

$R_G := \frac{N_G}{DP} \cdot \frac{1}{2}$       R<sub>G</sub> = 1.5       $R_P := \frac{N_P}{DP} \cdot \frac{1}{2}$       R<sub>P</sub> = 1

BL := Backlash\_lookup<sub>DP</sub>      BL = 0.01

$T_{P1} = T_{G1} = \frac{CP}{2} - \frac{BL}{2} = \frac{\pi}{DP \cdot 2} - \frac{BL}{2}$       allocate 1/2 backlash to each P & G       $CP := \frac{\pi \cdot 2 \cdot R_P}{N_P}$       CP = 0.314

$T_{P1} := \frac{\pi}{DP \cdot 2} - \frac{BL}{2}$       T<sub>P1</sub> = 0.152      T<sub>G1</sub> := T<sub>P1</sub>      T<sub>G1</sub> = 0.152      2 · T<sub>P1</sub> = 0.304

R<sub>root\_P</sub> := R<sub>P</sub> - dedendum      R<sub>root\_P</sub> = 0.875      root\_radius\_pinion      dedendum = 0.125

R<sub>root\_G</sub> := R<sub>G</sub> - dedendum      R<sub>root\_G</sub> = 1.375      root\_radius\_gear

R<sub>add\_P</sub> := R<sub>P</sub> + addendum      R<sub>add\_P</sub> = 1.1      addendum\_radius\_pinion      addendum = 0.1

R<sub>add\_G</sub> := R<sub>G</sub> + addendum      R<sub>add\_G</sub> = 1.6      addendum\_radius\_gear

inv(φ) := tan(φ) - φ      involute\_function

CTT<sub>1</sub> := T<sub>P1</sub>      circular\_tooth\_thickness\_at\_pitch\_radius

θ<sub>1</sub> := inv(φ<sub>1</sub>)      involute\_angle\_at\_pitch\_radius      θ<sub>1</sub> = 0.854 deg

geometry to determine points on involute between root and addendum  $R_2, B$

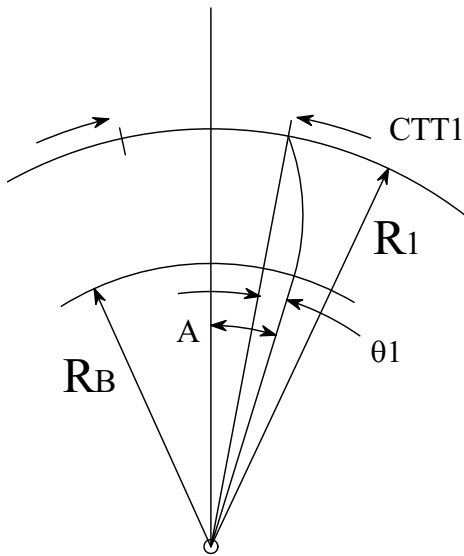


figure 2.10 page 31 Lynwander

reversed and rotated - values at pitch radius

$$A = \theta_1 + \frac{1}{2} \cdot \frac{CTT_1}{R_1}$$

$CTT_1$  = circular\_tooth\_thickness

$\phi$  = pressure\_angle\_design

$\theta_1$  = involute\_of\_design\_pressure\_angle

$$R_1 = \text{pitch\_radius} = \frac{R_B}{\cos(\phi)}$$

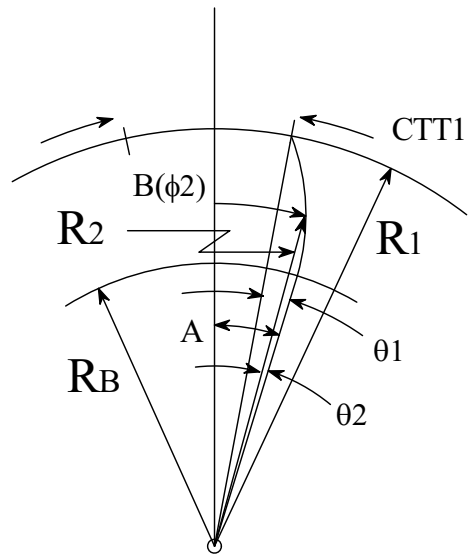


figure 2.10 page 31 Lynwander

reversed and rotated

here consider varying  $\phi$  from 0 to a value  $>$  design angle =  $\phi_2$

$\theta_2$  = involute\_of\_ $\phi_2$

$$B(\phi_2) = A - \theta_2$$

$$R_2 = \frac{R_B}{\cos(\phi_2)}$$

### Pinion geometry

$$\alpha := 0, 0.01 \dots 2 \cdot \pi$$

$\alpha$  = circular\_range\_variable

$$R_{B\_P} := R_P \cdot \cos(\phi_1)$$

base\_radius\_pinion

$$R_{B\_P} = 0.94$$

$$R_{\text{root\_P}} = 0.875$$

want to go from base to addendum radius in say 20 points

$$\phi_{\text{add\_P}} := \arccos\left(\frac{R_{B\_P}}{R_{\text{add\_P}}}\right)$$

$$\phi_{\text{add\_P}} = 31.321 \text{ deg}$$

$$R_{\text{root\_P}} > R_{B\_P} = 0$$

$$N1 := 20$$

$N1$  = number\_of\_points\_along\_involute

$$i := 1 \dots N1 + 1$$

$$\phi_{2i} := \frac{\phi_{\text{add\_P}}}{N1} \cdot (i - 1)$$

increment\_of\_pressure\_angle

let's put base radius to addendum in 1:n1+1

$$\theta_{2i} := \text{inv}(\phi_{2i})$$

involute\_angle\_at\_local\_radius

$$R_{2\_P_i} := \frac{R_{B\_P}}{\cos(\phi_{2i})}$$

radius\_on\_involute

$$CTT_{2_i} := 2 \cdot R_{2\_P_i} \cdot \left( \frac{CTT_1}{2 \cdot R_P} + \theta_1 - \theta_{2i} \right)$$

thickness\_at\_location

number of teeth  $j := 1 \dots N_P$   $B_{del_j} := (j - 1) \cdot \frac{2\pi}{N_P}$  angular increment for teeth (offset to angle B)

$B = \text{angle\_relative\_to\_tooth\_center} = \frac{\text{thickness}}{2 \cdot \text{radius\_at\_location}}$   $r, l = \text{right, left side of tooth}$

$Bl_{P_{i,j}} := \frac{CTT_{2_i}}{2 \cdot R_{2_{P_i}}} + B_{del_j}$   $Br_{P_{i,j}} := B_{del_j} - \frac{CTT_{2_i}}{2 \cdot R_{2_{P_i}}}$   $i = \text{range\_variable\_along\_involute}$   
 $j = \text{tooth\_number}$

adding a point at the root radius so we need to add two values of  $R_{root}$  and one each of  $Br$  and  $Bl$ . these are the first points

$R_{2_{P_0}} := R_{root\_P}$   $Bl_{P_{0,j}} := Bl_{P_{1,j}}$   $Br_{P_{0,j}} := Br_{P_{1,j}}$

now  $i$  needs to go from 0 to  $N1 + 1$   $N1 := N1 + 1$   $i := 0 \dots N1$

put into vector of  $R$  and  $\theta$  for polar plot effectiveness

$N1+2$  points from  $i$  up and down. radius up across then down across connecting the dots ...

$R_{plot\_P_{i+(N1+1) \cdot 2 \cdot (j-1)}} := R_{2_{P_i}}$   $R_{plot\_P_{i+(N1+1) \cdot [2 \cdot (j-1) + 1]}} := R_{2_{P_{N1-i}}}$

put right data first

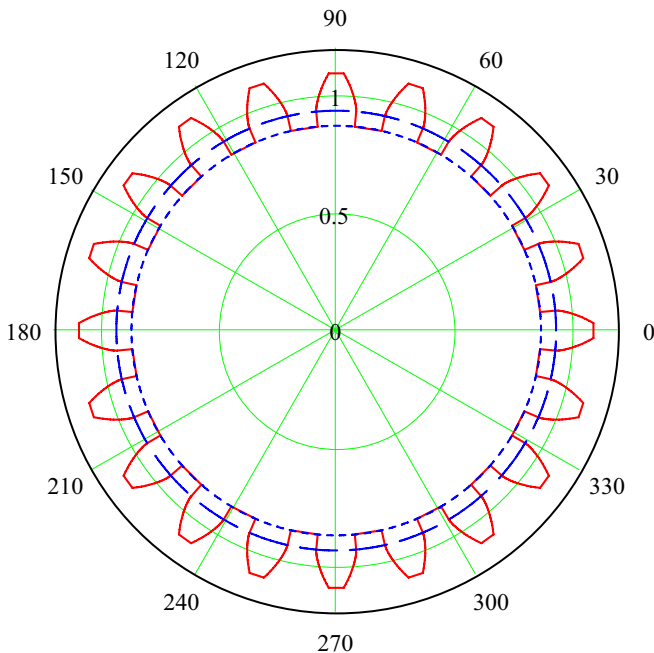
alternate by and "right" and "left" in sequence

$B_{plot\_P_{i+(N1+1) \cdot 2 \cdot (j-1)}} := Br_{P_{i,j}}$   $B_{plot\_P_{i+(N1+1) \cdot [2 \cdot (j-1) + 1]}} := Bl_{P_{N1-i,j}}$

bug := rows( $R_{plot\_P}$ )  $R_{plot\_P_{bug}} := R_{plot\_P_0}$   $B_{plot\_P_{bug}} := B_{plot\_P_0}$  close curve  $krhs := 1$

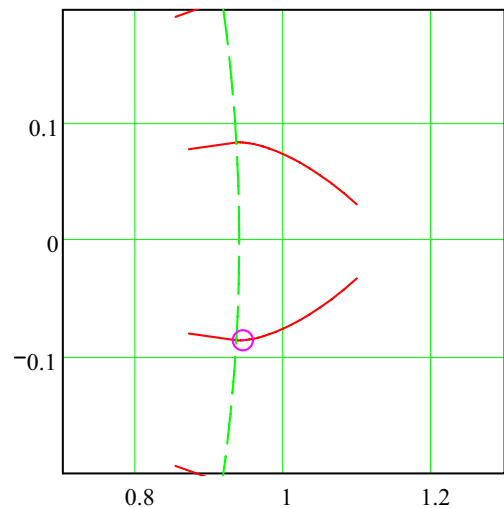
convert to X, Y coordinates to allow a closeup of one tooth, cannot restrain  $\theta$  in polar plot

$Xr_{P_{i,j}} := R_{2_{P_i}} \cdot \cos(Br_{P_{i,j}})$   $Xl_{P_{i,j}} := R_{2_{P_i}} \cdot \cos(Bl_{P_{i,j}})$   
 $Yr_{P_{i,j}} := R_{2_{P_i}} \cdot \sin(Br_{P_{i,j}})$   $Yl_{P_{i,j}} := R_{2_{P_i}} \cdot \sin(Bl_{P_{i,j}})$



— gear outline  
 - - - root radius  
 — base radius

closeup of tooth



## Gear geometry

$\phi_2 := \text{reset}$  reset  $\phi_2$  to avoid extra values in gear

$$R_{B\_G} := R_G \cdot \cos(\phi_1) \quad \text{base\_radius\_gear}$$

$$R_{B\_G} = 1.41 \quad N_2 := 24 \quad R_{\text{root\_G}} = 1.375$$

want to go from base to addendum radius in say 20 points retained separate number N2

$$\phi_{\text{add\_G}} := \arccos\left(\frac{R_{B\_G}}{R_{\text{add\_G}}}\right) \quad \phi_{\text{add\_G}} = 28.241 \text{ deg}$$

$$N_2 := 20 \quad N_2 = \text{number\_of\_points\_along\_involute}$$

$$R_{\text{root\_G}} > R_{B\_G} = 0$$

$$\phi_{\text{ded\_G}} := \arccos\left(\frac{R_{B\_G}}{R_{\text{root\_G}}}\right) \quad \phi_{\text{ded\_G}} = 12.816i \text{ deg}$$

$$\phi_{\text{root\_G}} := \text{if}(R_{\text{root\_G}} > R_{B\_G}, \phi_{\text{ded\_G}}, 0)$$

if root is > base, start involute at root not base. to allow the opposite, insert extra point as in pinion.

$$i := 1..N_2 + 1$$

$$\phi_{2i} := \phi_{\text{root\_G}} + \frac{\phi_{\text{add\_G}} - \phi_{\text{root\_G}}}{N_2} \cdot (i - 1) \quad \text{increment\_of\_pressure\_angle}$$

$$\theta_{2i} := \text{inv}(\phi_{2i})$$

involute\_angle\_at\_local\_radius

$$R_{2\_G} := \frac{R_{B\_G}}{\cos(\phi_{2i})} \quad \text{radius\_on\_involute}$$

$$CTT_{2i} := 2 \cdot R_{2\_G} \cdot \left( \frac{CTT_1}{2 \cdot R_G} + \theta_1 - \theta_{2i} \right) \quad \text{thickness\_at\_location}$$

$$j := 1..N_G \quad B_{\text{del},j} := (j - 1) \cdot \frac{2\pi}{N_G} \quad \text{angular increment for teeth (offset to angle B)}$$

$$B = \text{angle\_relative\_to\_tooth\_center} = \frac{\text{thickness}}{2 \cdot \text{radius\_at\_location}}$$

r,l = right,left side of tooth

$$Bl_{G,i,j} := \frac{CTT_{2i}}{2 \cdot R_{2\_G}} + B_{\text{del},j}$$

$$Br_{G,i,j} := B_{\text{del},j} - \frac{CTT_{2i}}{2 \cdot R_{2\_G}}$$

i = range\_variable\_along\_involute

j = tooth\_number

adding a point at the root radius,  $R_{\text{root}}$  is  $\max(R_B, R_{\text{root}})$  and one each of Br and Bl. these are the first points. in either case, added point is  $R_{\text{root\_G}}$ .

$$R_{2\_G} := R_{\text{root\_G}}$$

$$Bl_{G,0,j} := Bl_{G,1,j}$$

$$Br_{G,0,j} := Br_{G,1,j}$$

put into vector of R and  $\theta$  for polar plot effectiveness

$$\text{now } i \text{ needs to go from } 0 \text{ to } N_2 + 1 \quad N_2 := N_2 + 1 \quad i := 0..N_2$$

$N_2$  points from i up and down. radius up across then down across connecting the dots ...

$$R_{\text{plot\_G}_{i+(N_2+1) \cdot 2 \cdot (j-1)}} := R_{2\_G_i}$$

$$R_{\text{plot\_G}_{i+(N_2+1) \cdot [2 \cdot (j-1)+1]}} := R_{2\_G_{N_2-i}}$$

put right data first, alternate by and "right" and "left" in sequence

$$B_{\text{plot\_G}_{i+(N_2+1) \cdot 2 \cdot (j-1)}} := Br_{G_i,j}$$

$$B_{\text{plot\_G}_{i+(N_2+1) \cdot [2 \cdot (j-1)+1]}} := Bl_{G_{N_2-i},j}$$

$$\text{bug} := \text{rows}(R_{\text{plot\_G}})$$

$$R_{\text{plot\_G}}_{\text{bug}} := R_{\text{plot\_G}_0}$$

$$B_{\text{plot\_G}}_{\text{bug}} := B_{\text{plot\_G}_0}$$

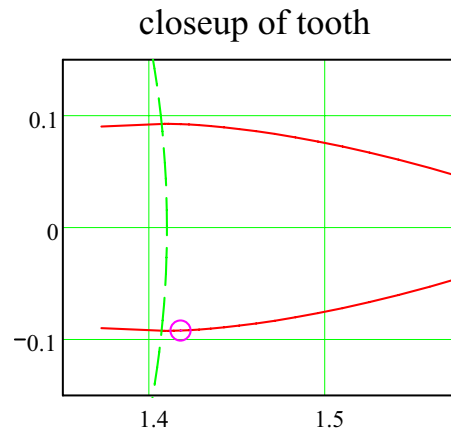
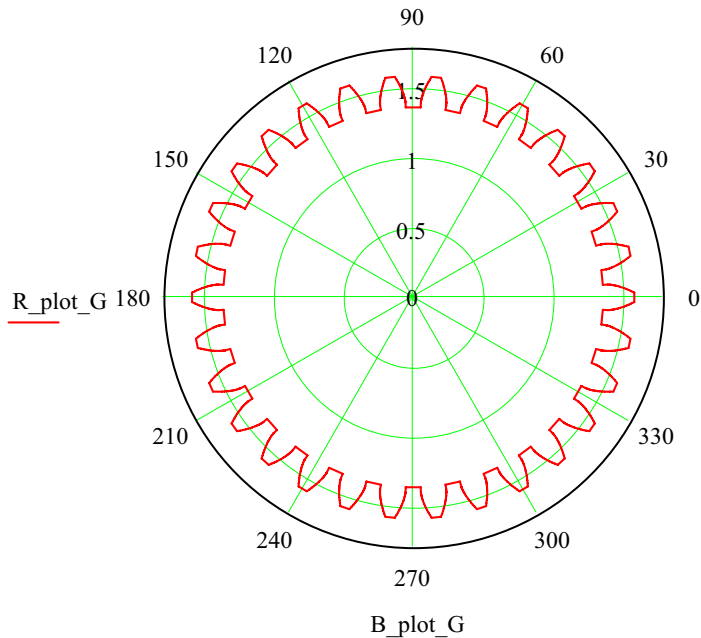
close curve

convert to X, Y coordinates to allow a closeup of one tooth, cannot restrain  $\theta$  in polar plot

$$krhs := 1$$

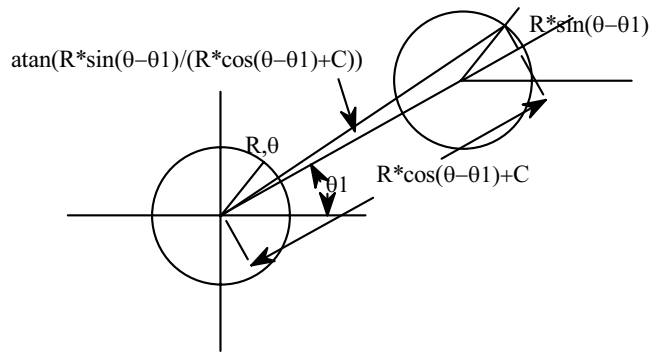
$$Xr_{G,i,j} := R_{2\_G_i} \cdot \cos(Br_{G_i,j}) \quad Xl_{G,i,j} := R_{2\_G_i} \cdot \cos(Bl_{G_i,j})$$

$$Yr_{G,i,j} := R_{2\_G_i} \cdot \sin(Br_{G_i,j}) \quad Yl_{G,i,j} := R_{2\_G_i} \cdot \sin(Bl_{G_i,j})$$



geometry to shift gear to appropriate center

relationships to shift a circle  $R, \theta$  from center at  $0,0$  to  $C,0$



$$R_{21}(R, \theta, C, \theta_1) := \sqrt{(R \cdot \cos(\theta - \theta_1) + C)^2 + (R \cdot \sin(\theta - \theta_1))^2} \quad \theta_1 = \text{angle\_circle\_center\_rotated}$$

$$\theta_{21}(R, \theta, C, \theta_1) := \left( \text{atan} \left( \frac{R \cdot \sin(\theta - \theta_1)}{R \cdot \cos(\theta - \theta_1) + C} \right) + \theta_1 \right)$$

shift gear a distance  $C$ , no rotation of center but rotate gear ( $B_p$ ) by  $1/2$  circular pitch angle to mesh

$$\text{Gear} \quad i := 0 \dots \text{rows}(R\_plot\_G) - 1 \quad B\_shift := \frac{\pi}{N_G} \quad B\_shift = 6 \text{ deg}$$

add rotation dependent on FRAME start at  $1/2$  -CP go to CP/2 ??

$$B\_rot := -B\_shift + 2 \cdot \frac{B\_shift}{100} \cdot \text{FRAME}$$

$$B\_rot = -6 \text{ deg}$$

and finally ... remove BL for meshing, applying half the distance on each of pinion and gear pinion is rotating CCW so adjust  $BL/4 \cdot R_p$  and gear is CW so add  $BL/4 \cdot R_G$  (CCW) to gear

$$B\_adj\_P := \frac{BL}{4 \cdot R_P}$$

$$B\_shift - B\_rot + B\_adj\_G = \text{total\_rotation\_of\_gear}$$

applied before translation

$$B\_adj\_G := \frac{BL}{4 \cdot R_G}$$

$$R_{plot\_G\_1_i} := R_{21}(R_{plot\_G_i}, B_{plot\_G_i} + B_{shift} - B_{rot} + B_{adj\_G}, C, 0)$$

$$B_{plot\_G\_1_i} := \theta_{21}(R_{plot\_G_i}, B_{plot\_G_i} + B_{shift} - B_{rot} + B_{adj\_G}, C, 0)$$

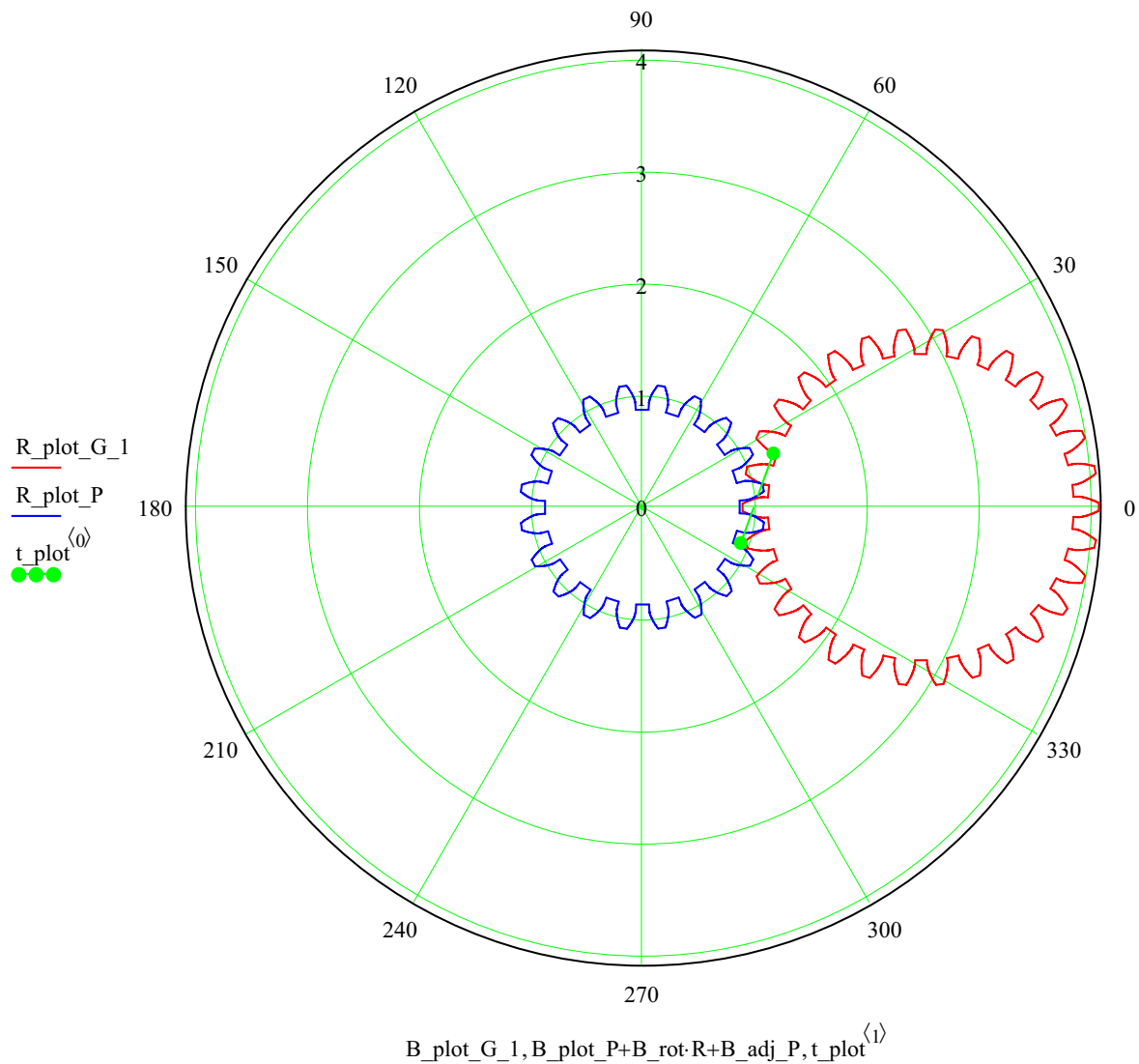
now add tangent to the mix ... pinion is rotating ccw therefore tangent is at:  $R_{B\_P}, -\phi_1$

gear is rotating cw therefore tangent before shift is at:  $R_{B\_G}, \pi - \phi_1$

and we need to translate it  $R_{tan\_G} := R_{21}(R_{B\_G}, \pi - \phi_1, C, 0)$   $\theta_{tan\_G} := \theta_{21}(R_{B\_G}, \pi - \phi_1, C, 0)$

so tangent plot is

$$t_{plot} := \begin{pmatrix} R_{B\_P} & -\phi_1 \\ R_{tan\_G} & \theta_{tan\_G} \end{pmatrix} \quad t_{plot} = \begin{pmatrix} 0.94 & -0.349 \\ 1.27 & 0.389 \end{pmatrix}$$



X\_G := reset    Y\_G := reset    shift to X,Y so can get closeup of mesh in animation    j := 0 .. rows(R\_plot\_P) - 1

$$X_{G_i} := R_{plot\_G\_1_i} \cdot \cos(B_{plot\_G\_1_i})$$

$$X_{P_j} := R_{plot\_P_j} \cdot \cos(B_{plot\_P_j} + B_{rot} \cdot R + B_{adj\_P})$$

$$Y_{G_i} := R_{plot\_G\_1_i} \cdot \sin(B_{plot\_G\_1_i})$$

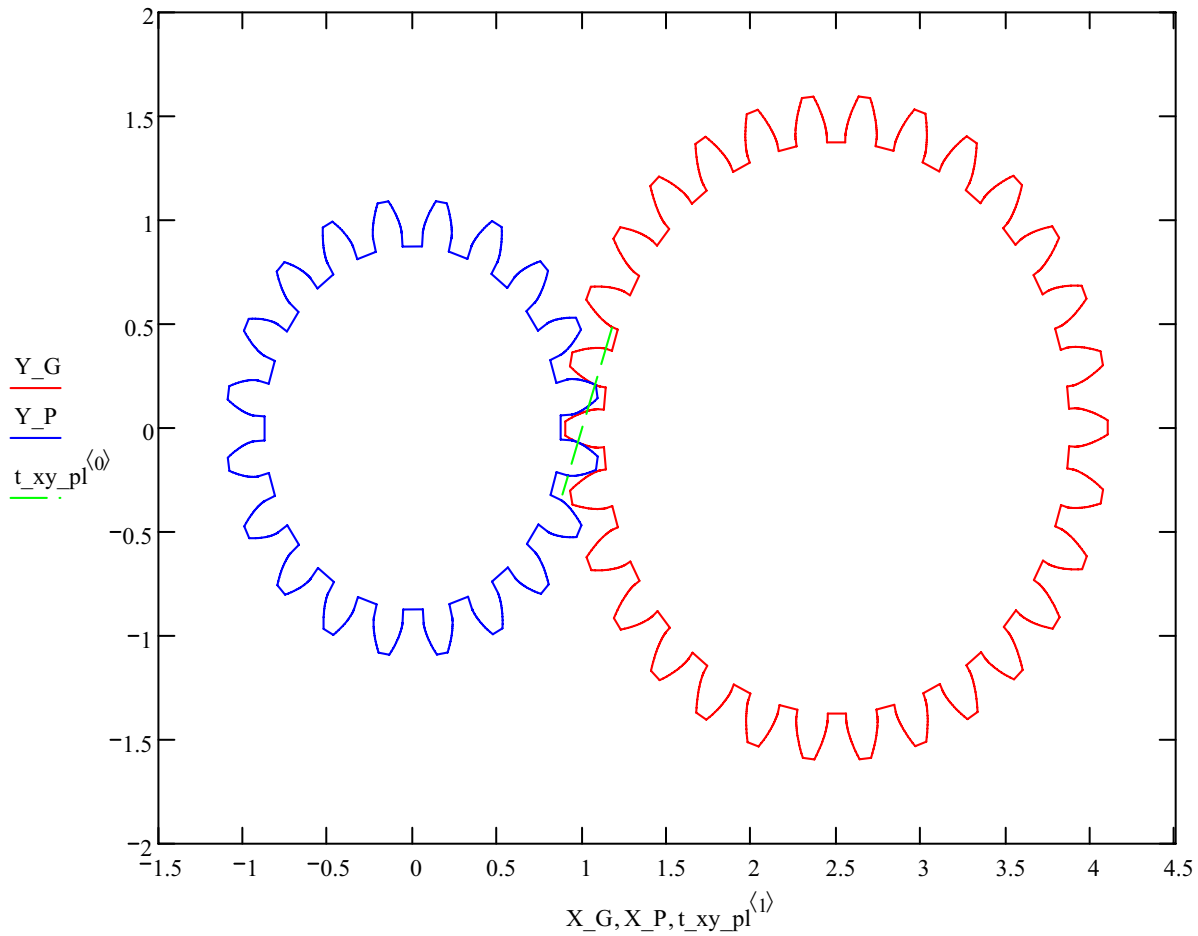
$$Y_{P_j} := R_{plot\_P_j} \cdot \sin(B_{plot\_P_j} + B_{rot} \cdot R + B_{adj\_P})$$

$$X_{tan\_G} := R_{tan\_G} \cdot \cos(\theta_{tan\_G})$$

$$X_{tan\_P} := R_{B\_P} \cdot \cos(-\phi_1)$$

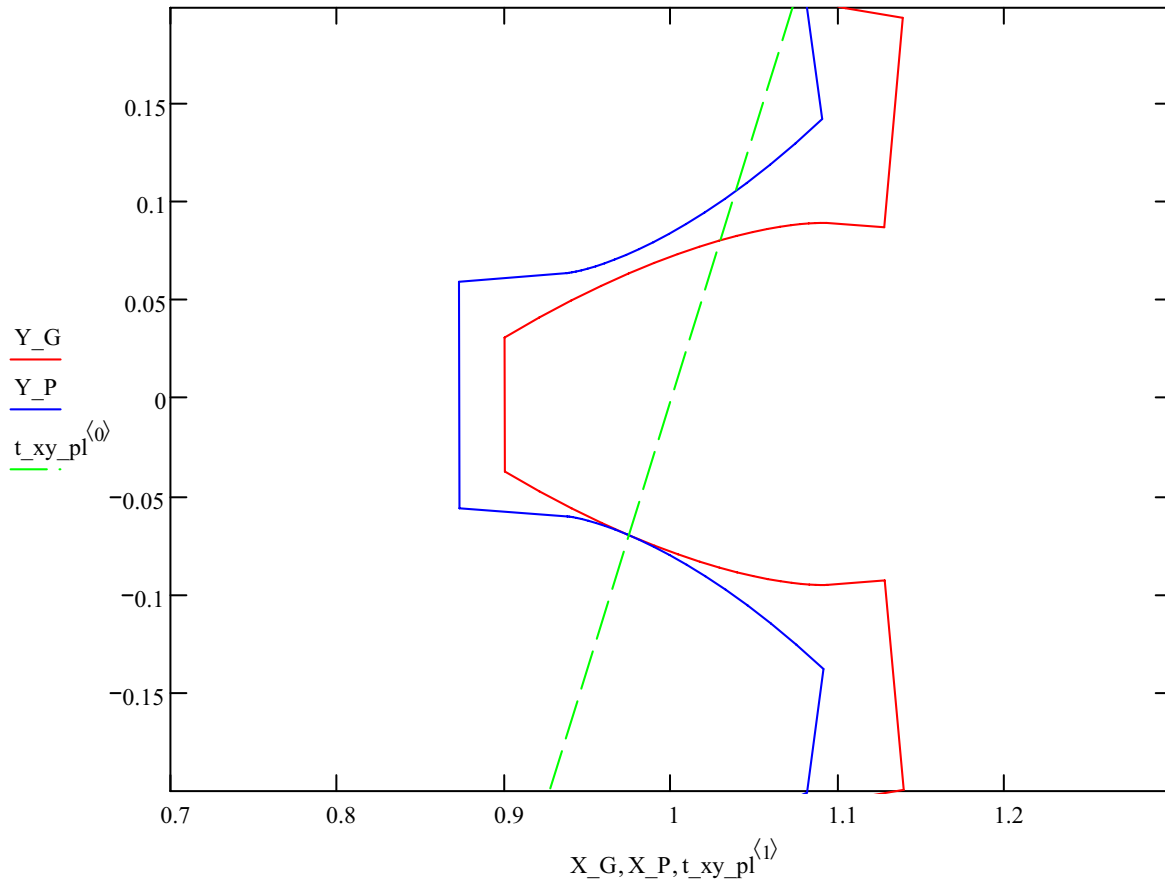
$$Y_{tan\_G} := R_{tan\_G} \cdot \sin(\theta_{tan\_G})$$

$$Y_{tan\_P} := R_{B\_P} \cdot \sin(-\phi_1) \quad t_{xy\_pl} := \begin{pmatrix} Y_{tan\_G} & X_{tan\_G} \\ Y_{tan\_P} & X_{tan\_P} \end{pmatrix}$$



B\_rot · R = -9 deg

R = 1.5    N<sub>P</sub> = 20    N<sub>G</sub> = 30



These last two figures are animated in gear mesh video revised. In animating, the variable FRAME is incremented from 0 : 100, the calculations highlighted above are carried out and plotted, the plots updated and a video screen captured.