LQG/LTR design: term project. You are asked to carry out modeling and control system design for the surge dynamics of an oceanic cable-laying vessel. This vessel needs to have very good speed control so that the cable, which is being paid out constantly and sinks slowly under its own weight, lies properly on the seafloor. The bathymetry for the path is well-known but variable, so the vessel will need to slow down and speed up frequently. *Unexpected* deviations in speed from the calculated trajectory are likely to create loops or kinks in the cable, or induce large tensions.

vessel	length	L	$145m$
	draft	h	$9m$
	beam	b	$22m$
	displacement	∇	$17750m^3$
	surge added mass	m_a	$0.07\rho\nabla$
	thrust reduction factor	t	0.19
	wetted surface area	A_w	$5585m^3$
	towed resistance coefficient	C_r	0.0025
	wake fraction at propeller	w	0.22
propeller	diameter	D	$7.21m$
	pitch/diameter ratio	P/D	1.1
	rotative efficiency	η_r	1.025
	zero-speed thrust coefficient	$k_t(J=0)$	0.6
	slope of k_t with J		-0.522
	zero-speed torque coefficient	$k_q(J=0)$	0.1
	slope of k_q with J		-0.0833
	rotational moment of inertia	I_p	$2.03 \times 10^5 kgm^2$
engine	parameter	a	0.876
		b	0.208
		c	2.173
		d	$-.0939$
	maximum fuel rate	f_m	$1.5 kg/s$
	maximum torque	Q_m	$70000 Nm$
	maximum speed	n_m	$60 Hz$
gearbox	reduction ratio	λ	32
	efficiency	η_g	0.97

It is desired to create a linear controller for the nominal surge motion, and then demonstrate that it will work with a simulation of the real nonlinear system, within a neighborhood of the nominal speed.

The vessel is powered by a single gas turbine engine, driving one propeller. The parameters for the vessel, propeller, and engine system are as given in the table.

The gas turbine torque-speed characteristic fits the relation:

$$\frac{Q_e}{Q_m} = -\left(a\frac{f}{f_m} + b\right)\frac{n_e}{n_m} + \left(c\frac{f}{f_m} + d\right),$$

where Q_e represents the engine torque in Nm, f the fuel rate in kg/s, and n_e the rotation rate of the engine in Hz.

1. Make a map of the gas turbine characteristics. For instance, make a contour plot which gives Q_e/Q_m (the fraction of maximum torque developed) as a function of f/f_m (the fraction of the maximum fuel rate) and n_e/n_m (the fraction of the maximum rotation rate of the engine).

   Be sure to include clipping on your contours, where the calculated torque exceeds Q_m. Similarly, the engine cannot develop negative torque.

2. Make a table of some steady operating conditions for f_o/f_m in the range of 0.05-0.95, in which you show: advance ratio seen by the propeller, engine torque, propeller rotational speed, engine rotational speed, and vessel linear speed.

   Plot the pairs [f_o/f_m, n_{eo}/n_m] on the characteristic plot from Part 1, and assess whether the vessel, engine, and propeller are well matched to operate over a range of f_o.

3. Construct a linear approximation for the plant dynamics around the operating condition $f_o/f_m = 0.8$. If the plant input is δf, and the output is δu, list the A, B, C matrices for your plant, and the eigenvalues. δ here indicates the perturbed value from the steady state, e.g., $f = f_o + \delta f$.

4. Create an LTR-type controller for the plant. First, choose the filter gains V_1 and V_2 to make the crossover frequency (where the magnitude of $L(s) = C(sI - A)^{-1}H$ passes through unity) about 0.6 rad/s. This choice corresponds with the closed-loop system being able to overcome waves with period 10 $seconds$ or longer. Note that for this scalar design, you will be able to move the curve $|L(s)|$ up or down on the Bode plot, but you cannot easily change its shape, or move it side-to-side.

   Secondly, recover this loop shape with an LQG-type controller, using a small control penalty.

   You should prepare for this part: a plot of $log(|L(s)|)$ versus $log(\omega)$, a listing of your V_1 and V_2 choices, and the A, B, C matrices of your LQG compensator.

5. In the event of a current or wind disturbance, the actual vessel speed will vary from the nominal value. This is not acceptable in the long term, given that the vessel is laying out a cable.

   An integral action can be added quite easily in the LTR design technique. First, add an integrator to the input of the plant, so that an ``augmented plant'' is created. This augmented plant has one more state than the original plant:

$$A_{aug} = \begin{bmatrix} A & B \\ 0 & 0 \end{bmatrix}$$
$$B_{aug} = [0 \ 1]^T$$
$$C_{aug} = [C \ 0]$$

   The idea is to then carry out the KF design and LTR as before, with the augmented plant model. When this is done, move the integrator from the augmented plant into the compensator. The addition of an integrator channel to the plant or compensator

can be accomplished with the command `sysPaug = sysP * tf([0 1],[1 0]) ;`, for example.

Present a figure of $|L(s)|$ and the other quantities as you gave them in Part 4. This is to be kept as a separate controller design.

6. For the purpose of demonstrating your controllers, construct a nonlinear simulation of the plant system, with input f. You don't need to show anything for this task; it is mostly to verify that the steady conditions you calculated above, with $f = 0.8 f_m$, are actually steady conditions from the point of view of the whole simulation. You may wish to use `global` variables in Matlab so that you don't have to type everything in more than once.

7. Augment your simulation above with the compensator dynamics. If the compensator has states z, and your simulation has states $[u, n_p]$, make a new state vector $[u, n_p, z]$. You will propagate the first two states as usual with the nonlinear equations and the fuel rate f as input. The compensator states z are propagated by a set of system matrices $[A_c, B_c, C_c]$, which you just designed. Note that the input to the compensator is an error signal: $e = r - u$, where u is the speed of the vessel and r is a reference speed. The output of the compensator is then fuel rate f.

We are interested mainly in disturbance rejection, for which you will just set $r = u_0$, the steady condition speed. Recall that you are implementing a controller for operation about a nominal condition; whenever your compensator generates f from the error signal, be sure to *then* add on $0.8 f_m$, the part needed to keep us near the steady condition.

Demonstrate four properties, showing time-domain simulations for each:

- The first controller, without integral action, does not completely reject steady disturbances. I suggest you implement a disturbance as an acceleration:
  `uprime = uprime + k`. In the absence of a controller, this line would drive the system with an additional acceleration of $k\ m/s^2$, thus modelling the effects of steady wind or current. This line is to be added after computing the usual parts of `uprime`.
- The first controller does not reject high frequency disturbances. A suggestion is to let
  `uprime = uprime + k*sin(10*omega_c*t)`, where `omega_c` is the crossover frequency. This adds an oscillating acceleration at ten times the crossover frequency.
- The first controller does, however, reject some lower frequency disturbances. For example, try
  `uprime = uprime + k*sin(omega_c/10*t)/100`. Note the extra factor of 100 is included so that the result can be compared directly with the high-frequency disturbance case above. The effect of an acceleration disturbance onto the observed output, velocity u, scales with the inverse of the frequency.
- The second controller, with integral action, rejects steady disturbances. Use the same approach as above for steady disturbances

8. What is the propeller response during the high-frequency disturbances? Is it reasonable or do we need to slow down the controller bandwidth?