2.092/2.093

FINITE ELEMENT ANALYSIS OF SOLIDS AND FLUIDS I

FALL 2009

Quiz #1-solution

Instructor: Prof. K. J. Bathe
TA: Seounghyun Ham

Problem 1 (10 points):

a)

$$u^{(1)}(x)=\underline{H}^{(1)}\underline{U}=\begin{bmatrix}1-\frac{x}{60} & \frac{x}{60} & 0\end{bmatrix}\underline{U}$$

$$u^{(2)}(x)=\underline{H}^{(2)}\underline{U}=\begin{bmatrix}0 & 1-\frac{x}{100} & \frac{x}{100}\end{bmatrix}\underline{U}$$

where $\underline{U}=\begin{bmatrix}U_1 & U_2 & U_3\end{bmatrix}^T$

Therefore, $\underline{H}^{(1)}=\begin{bmatrix}1-\frac{x}{60} & \frac{x}{60} & 0\end{bmatrix}$

$$\underline{H}^{(2)}=\begin{bmatrix}0 & 1-\frac{x}{100} & \frac{x}{100}\end{bmatrix}$$

b)

$$\varepsilon^{(1)}(x)=\begin{bmatrix}\frac{\partial u}{\partial x}\end{bmatrix}=\begin{bmatrix}\frac{\partial}{\partial x}\end{bmatrix}\underline{H}^{(1)}\underline{U}=\begin{bmatrix}-\frac{1}{60} & \frac{1}{60} & 0\end{bmatrix}\underline{U}=\underline{B}^{(1)}\underline{U}$$

$$\varepsilon^{(2)}(x)=\begin{bmatrix}\frac{\partial u}{\partial x}\end{bmatrix}=\begin{bmatrix}\frac{\partial}{\partial x}\end{bmatrix}\underline{H}^{(2)}\underline{U}=\begin{bmatrix}0 & -\frac{1}{100} & \frac{1}{100}\end{bmatrix}\underline{U}=\underline{B}^{(2)}\underline{U}$$

$$\underline{K}^{(1)} = \int_{V^{(1)}} \underline{B}^{(1)T}\underline{C}\underline{B}^{(1)}dV^{(1)} = \int_{x=0}^{x=60} \underline{B}^{(1)T}\underline{C}\underline{B}^{(1)}A^{(1)}(x)dx = \int_{x=0}^{x=60} \begin{bmatrix} -\frac{1}{60} \\ \frac{1}{60} \\ 0 \end{bmatrix} E \begin{bmatrix} -\frac{1}{60} & \frac{1}{60} & 0 \end{bmatrix} (4-\frac{x}{20})^2 dx$$

$$\underline{K}^{(2)} = \int_{V^{(2)}} \underline{B}^{(2)T}\underline{C}\underline{B}^{(2)}dV^{(2)} = \int_{x=0}^{x=100} \underline{B}^{(2)T}\underline{C}\underline{B}^{(2)}A^{(2)}(x)dx = \int_{x=0}^{x=100} \begin{bmatrix} 0 \\ -\frac{1}{100} \\ \frac{1}{100} \end{bmatrix} E \begin{bmatrix} 0 & -\frac{1}{100} & \frac{1}{100} \end{bmatrix} (1)dx$$

$$\underline{R}^{(1)} = \underline{R}^{B(1)} = \int_{V^{(1)}} \underline{H}^{(1)T} f_x^{B(1)} dV^{(1)} = \int_{x=0}^{x=60} \underline{H}^{(1)T} f_x^{B(1)} A^{(1)}(x)dx = \int_{x=0}^{x=60} \begin{bmatrix} 1-\frac{x}{60} \\ \frac{x}{60} \\ 0 \end{bmatrix} (\rho\omega^2 x)(4-\frac{x}{20})^2 dx$$

$$\underline{R}^{(2)} = \underline{R}^{B(2)} = \int_{V^{(2)}} \underline{H}^{(2)T} f_x^{B(2)} dV^{(2)} = \int_{x=0}^{x=100} \underline{H}^{(2)T} f_x^{B(2)} A^{(2)}(x)dx = \int_{x=0}^{x=100} \begin{bmatrix} 0 \\ 1-\frac{x}{100} \\ \frac{x}{100} \end{bmatrix} (\rho\omega^2 (x+60))(1)dx$$

Problem 2 (10 points):

a)

$$h_1=\frac{1}{4}\left(1+\frac{x}{3}\right)\left(1+\frac{y}{2}\right)$$

$$h_2=\frac{1}{4}\left(1-\frac{x}{3}\right)\left(1+\frac{y}{2}\right)$$

$$h_3=\frac{1}{4}\left(1-\frac{x}{3}\right)\left(1-\frac{y}{2}\right)$$

$$h_4=\frac{1}{4}\left(1+\frac{x}{3}\right)\left(1-\frac{y}{2}\right)$$

$$\underline{u}=\begin{bmatrix} u(x,y) \\ v(x,y) \end{bmatrix}=\underline{H}\underline{U}=\begin{bmatrix} h_1 & h_2 & h_3 & h_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & h_1 & h_2 & h_3 & h_4 \end{bmatrix}\underline{U}$$

where $\underline{U}=\begin{bmatrix} u_1 & u_2 & u_3 & u_4 & v_1 & v_2 & v_3 & v_4 \end{bmatrix}^T$

Therefore,

b)

$$\underline{\varepsilon}=\begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{bmatrix}=\begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix}\underline{u}=\begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix}\underline{H}\underline{U}=\begin{bmatrix} h_{1,x} & h_{2,x} & h_{3,x} & h_{4,x} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & h_{1,y} & h_{2,y} & h_{3,y} & h_{4,y} \\ h_{1,y} & h_{2,y} & h_{3,y} & h_{4,y} & h_{1,x} & h_{2,x} & h_{3,x} & h_{4,x} \end{bmatrix}\underline{U}=\underline{B}\underline{U}$$

$$h_{1,x}=\frac{1}{12}\left(1+\frac{y}{2}\right) \quad , \quad h_{1,y}=\frac{1}{8}\left(1+\frac{x}{3}\right)$$

$$h_{2,x}=-\frac{1}{12}\left(1+\frac{y}{2}\right) \quad , \quad h_{2,y}=\frac{1}{8}\left(1-\frac{x}{3}\right)$$

$$h_{3,x}=-\frac{1}{12}\left(1-\frac{y}{2}\right) \quad , \quad h_{3,y}=-\frac{1}{8}\left(1-\frac{x}{3}\right)$$

$$h_{4,x}=\frac{1}{12}\left(1-\frac{y}{2}\right) \quad , \quad h_{4,y}=-\frac{1}{8}\left(1+\frac{x}{3}\right)$$

$$\underline{K}=\int_V \underline{B}^T\underline{C}\underline{B}dV=t\int_{y=-2}^{y=2}\int_{x=-3}^{x=3}\underline{B}^T\underline{C}\underline{B}dxdy$$

$$\underline{R}=\int_V \underline{H}^T\underline{f}^B dV=t\int_{y=-2}^{y=2}\int_{x=-3}^{x=3}\begin{bmatrix} h_1 & 0 \\ h_2 & 0 \\ h_3 & 0 \\ h_4 & 0 \\ 0 & h_1 \\ 0 & h_2 \\ 0 & h_3 \\ 0 & h_4 \end{bmatrix}\begin{bmatrix} 4+x \\ 0 \end{bmatrix}dxdy=t\int_{y=-2}^{y=2}\int_{x=-3}^{x=3}\begin{bmatrix} h_1(4+x) \\ h_2(4+x) \\ h_3(4+x) \\ h_4(4+x) \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}dxdy$$

2.092 / 2.093 Finite Element Analysis of Solids and Fluids I
Fall 2009