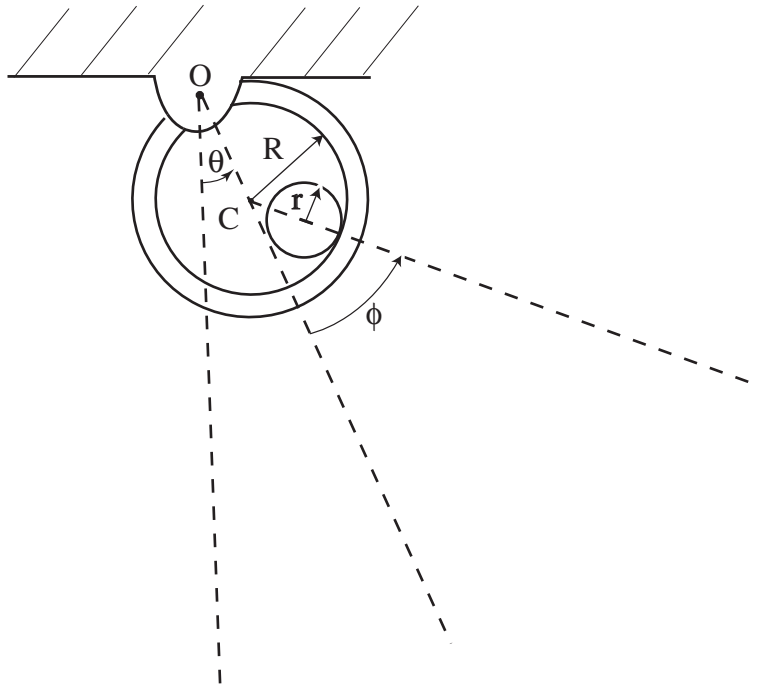**Practice Problems No.1**

Problem 1

A ring of radius R is pivoted without friction at O. A disk of radius r rolls without slipping inside the ring, as shown below. Determine the angular velocities of the ring and the disk in terms of the generalized coordinates θ, ϕ indicated.
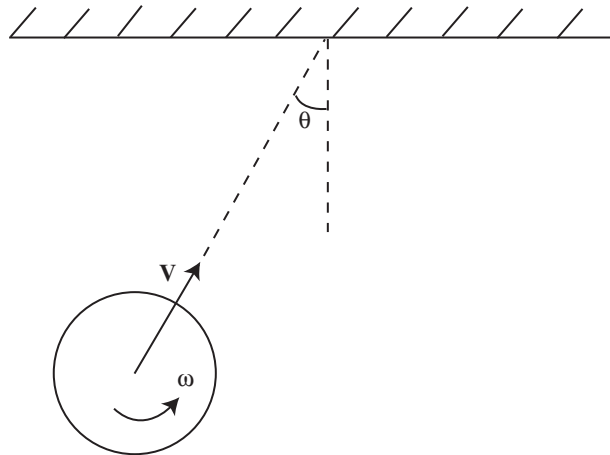

Problem 2

A thin rectangular plate of mass M, sides a and b, rotates about an axis along its diagonal with angular velocity ω.

(a) What are the forces on the bearings?

(b) What is the kinetic energy of the rotating plate?

Problem 3 (adapted from Doctoral Qualifying Exam 2000)

A rigid, uniform flat disk of mass m and radius R is moving in the plane towards a wall with central velocity $\boldsymbol{V}$ while rotating with angular velocity ω, as shown below. Assuming that the collision in the normal direction is elastic and no slip occurs at the wall, find the velocity of the (center of the) disk after it collides with the wall.

Problem 4 (adapted from Doctoral Qualifying Exam 2003)

The homogeneous sphere of mass m and radius r is projected along the incline of angle θ with an initial central velocity $\boldsymbol{V_0}$ and no angular velocity ($\omega_0 = 0$). If the coefficient of kinetic friction is μ, determine the time duration T of the period of slipping. In addition, state the velocity $\boldsymbol{V_f}$ of the mass center C at the end of the period of slipping.
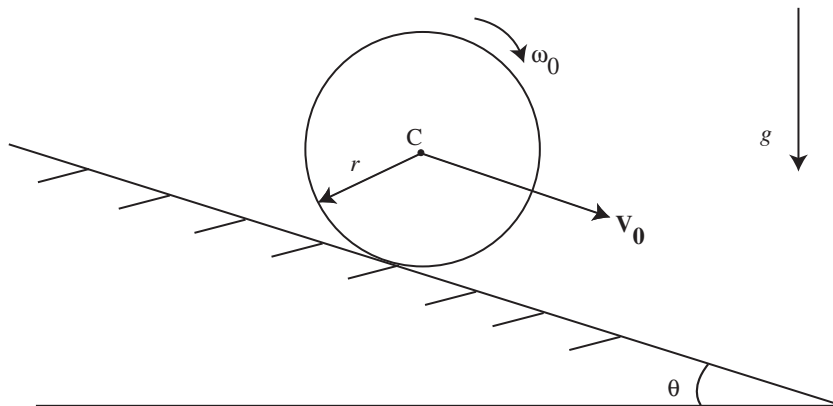
Problem 5

A pencil, modeled as a uniform bar AB of length $2a$ and mass m, is initially at rest standing upright at the edge of a table as shown below. A horizontal impulse ΔP is delivered to the base A of the pencil, causing it to lose contact with the table. Determine the value of ΔP for which the motion of the pencil is such that point B clips the edge of the table. Sketch the position of the pencil at this instant.