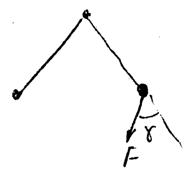
[redacted]

Session 11



F: follower force

$$F = -\nabla V$$

$$\Leftarrow \begin{cases} F_x = -\frac{\partial V}{\partial x} \\ F_y = -\frac{\partial V}{\partial y} \end{cases}$$

$$\frac{\partial}{\partial y} F_x = \frac{\partial}{\partial x} F_y$$

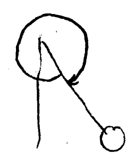
↳ F is not potential

works done by F

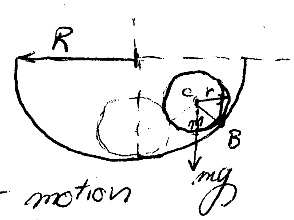
$$W_{12} = |F| \sin\gamma \, L(\varphi_2 - \varphi_1)$$

Note W_{12} is not path-independent

This system is locally Conservative



Example



1 DOF = 3-2=1 → choose φ as a generalized Coordinate

Question. Eq of motion
Reaction forces

To eliminate the role of Constraint forces, take any momentum principle w.r.t. B

$$\underline{\dot{H}}_B + \underline{V}_B \times \underline{P} = \underline{M}_B$$

$$\underline{M}_B = mgr \sin\varphi \, \underline{K}$$

$$\underline{V}_B \parallel \underline{V}_C \Rightarrow \underline{V}_B \parallel \underline{P}$$

$$H_B \neq \underline{\underline{I}}_B \underline{\omega} \,!$$

$$\Rightarrow \underline{H}_B = \underline{H}_C + \underline{P} \times \underline{r}_{CB}$$

$$= I_C \, \omega \, \underline{K} + \underline{P} \times \underline{r}_{CB}$$

$$= \frac{1}{2} m r^2 (-\dot{\theta}) \underline{K} + m(R-r)\dot{\varphi} \, (\cos\varphi \, \hat{i} + \sin\varphi \, \hat{j}) \times r(\sin\varphi \, \hat{i} - \cos\varphi \, \hat{j})$$

Note: $R\varphi = r(\theta + \varphi) \Rightarrow \dot{\theta} = \frac{R-r}{r}\dot{\varphi}$

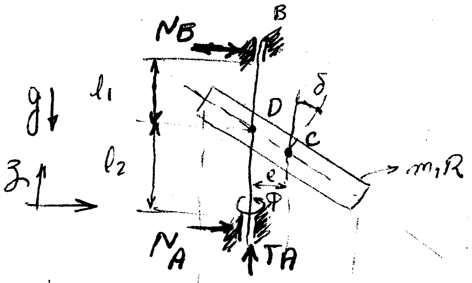
$$\Rightarrow \underline{H}_B = -\frac{3}{2} m r (R-r) \dot{\varphi} \, K$$

$\Rightarrow$ Eq. of motion: $-\frac{3}{2} m r (R-r) \ddot{\varphi} = mgr \sin\varphi \Rightarrow \ddot{\varphi} + \frac{2g}{3(R-r)} \sin\varphi = 0$

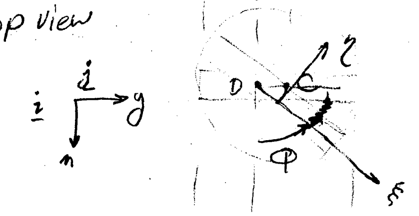
For frequency of small oscillations linearized → $\ddot{\varphi} + \frac{2g}{3(R-r)} \varphi = 0 \qquad \alpha = \sqrt{\frac{2g}{3(R-r)}}$

Example

eccentric skewed disk on a rotating shaft



top view



Reaction forces at A & B?

$\# DOF = 6-3-2=1 \rightarrow$ use φ

Linear momentum principle:

$$\dot{P} = m a = (N_A+N_B)\cos\varphi\,\hat{i} + (N_A+N_B)\sin\varphi\,\hat{j} + (T_A - Mg)\hat{k}$$

$$r_c = e(\cos\varphi\,\hat{i} + \sin\varphi\,\hat{j}) + z_c\,\hat{k}$$

$$\Rightarrow a_c = -e(\ddot{\varphi}\sin\varphi + \dot{\varphi}^2\cos\varphi)\hat{i} + e(\ddot{\varphi}\cos\varphi - \dot{\varphi}^2\sin\varphi)\hat{j}$$

$\Rightarrow$ linear momentum principle:

$$-em(\ddot{\varphi}\sin\varphi + \dot{\varphi}^2\cos\varphi) = (N_A+N_B)\cos\varphi$$
$$em(\ddot{\varphi}\cos\varphi - \dot{\varphi}^2\sin\varphi) = (N_A+N_B)\sin\varphi$$
$$0 = T_A - mg \longrightarrow \boxed{T_A = Mg}$$

$$\Rightarrow -em\dot{\varphi}^2 = N_A + N_B$$

Angular momentum principle: $\dot{H}_c + v_c \times P = M_c$

$$M_c = N_B(l_1 + e\tan\delta) - N_A(l_2 - e\tan\delta) + T_A e$$

$$\dot{H}_c = \overset{\circ}{H}_c + \omega \times H_c$$

relative to frame

$$H_c = I_c\,\omega = \begin{pmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{pmatrix}\begin{pmatrix} -\dot{\varphi}\sin\delta \\ 0 \\ \dot{\varphi}\cos\delta \end{pmatrix}$$

$$I_1 = I_2 = \frac{1}{4} m\left[R^2 + \frac{h^2}{3}\right], \quad I_3 = \frac{1}{2} mR^2$$

$$= \begin{Bmatrix} -I_1\dot{\varphi}\sin\delta \\ 0 \\ I_3\dot{\varphi}\cos\delta \end{Bmatrix} \Rightarrow \dot{H}_c = \overset{\circ}{H}_c + \omega\times H_c = \begin{Bmatrix} -I_1\ddot{\varphi}\sin\delta \\ 0 \\ I_3\ddot{\varphi}\cos\delta \end{Bmatrix} + \begin{vmatrix} e_1 & e_2 & e_3 \\ -\dot{\varphi}\sin\delta & 0 & \dot{\varphi}\cos\delta \\ -I_1\dot{\varphi}\sin\delta & 0 & I_3\dot{\varphi}\cos\delta \end{vmatrix}$$

$$= \begin{pmatrix} -I_1\ddot{\varphi}\sin\delta \\ -\dot{\varphi}^2\sin\delta\cos\delta(I_1 - I_3) \\ I_3\ddot{\varphi}\cos\delta \end{pmatrix}$$