2.004 Dynamics and Control II
Spring 2008

Massachusetts Institute of Technology
Department of Mechanical Engineering

Dynamics and Control II
Spring Term 2008

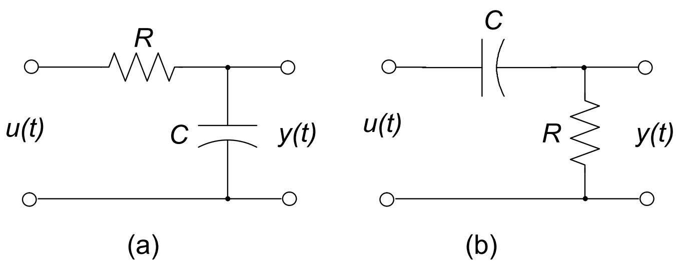
Problem Set 10

Assigned: May 2, 2008 Due: May 9, 2008

Reading:

- Nise Secs. 10.1, 10.2
- Class Handout: Sinusoidal Frequency Response of Linear Systems

Problem 1: In the processing of audio signals, it is often desired to amplify or attenuate signals in a given frequency range (bass and/or treble boost/cut). Two electrical circuits are shown below are identified as either low-pass (transmitting low frequency signals while attenuating high frequencies), or high-pass (transmitting high frequency signals and attenuating low frequencies).



(a) Derive the transfer functions between the input and output voltages for each circuit.

(b) Plot the pole-zero plots for each circuit.

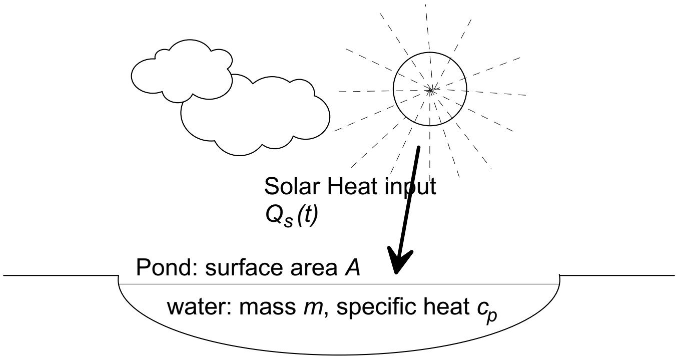
(c) Sketch the amplitude and phase of the frequency response functions for the circuits when $R = 10,000\ \Omega$, and $C = 1\ \mu$F.

(d) Identify which circuit is low-pass and which is high-pass?

Problem 2: The solar pond shown below is used to store energy in the form of hot water. Assume that the system input is $Q_s(t)$, the solar heat input and that the output is the pond temperature T. Measurements and analysis have shown that it is reasonable to model the pond as a first order system with the transfer function:

$$H(s) = \frac{T(s)}{Q_s(s)} = \frac{1/(mc_p)}{s + (hA/(mc_p))}$$

where m is the mass of the water, c_p is the specific heat, A is the surface area of the pond and h is the heat transfer coefficient.



(a) Use the transfer function to write a differential equation relating the pond temperature to the solar input.

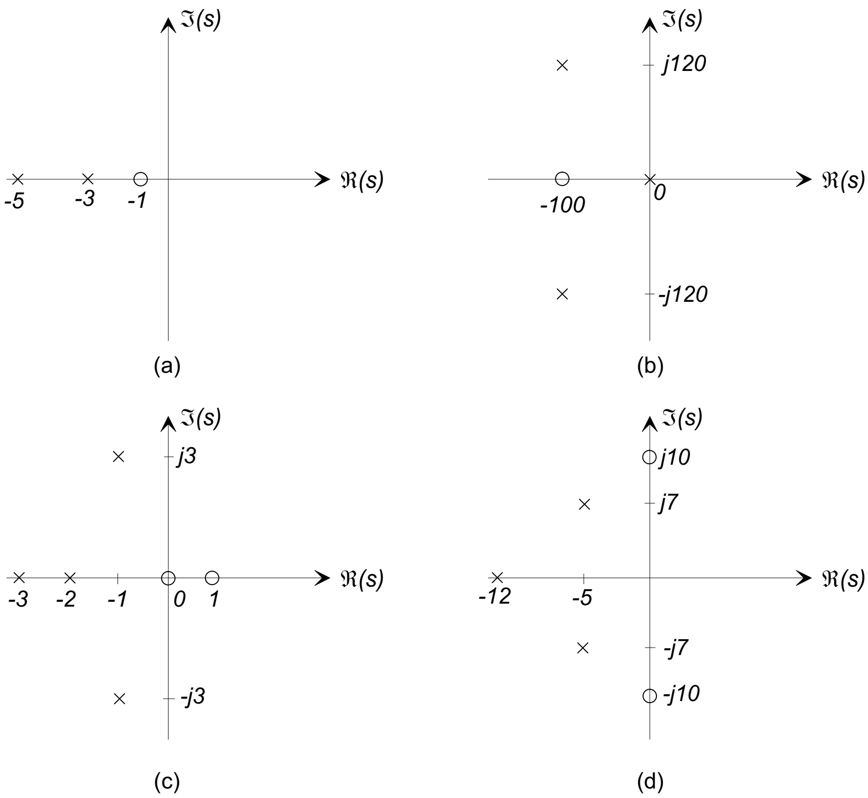
(b) If the solar heat flow is a constant, that is $Q_s(t) = Q_o$, find the steady-state temperature that the pond will reach in terms of system parameters.

(c) Determine the magnitude and phase of the system frequency response.

(d) Now assume that the solar flux undergoes annual seasonal variations about a mean value that is $Q_s(t) = Q_o \sin(\omega t - \pi/2) + Q_{avg}$ where $\omega = 2\pi/365$ rad/day, and the phase shift of $-\pi/2$ ensures maximum solar flux in the summer. What is the annual fluctuation, maximum to minimum of the pond temperature?

(e) What day of the year does the pond reach its maximum temperature?

Problem 3: Four systems have the pole-zero plots shown below.



For each system determine from the pole-zero plot

(a) the slope of the high frequency magnitude asymptote,

(b) the asymptotic high frequency phase response,

(c) the low frequency asymptotic magnitude behavior, and

(d) the low frequency phase shift.

Problem 4: A *non-minimum phase* linear system is defined as one with one or more zeros in the right-half of the s-plane.

(a) Does a non-minimum phase zero affect system stability?

(b) Construct pole-zero plots and Bode plots for the following two systems:

$$H_1(s) = \frac{s+a}{s+b}, \qquad H_2(s) = -\frac{s-a}{s+b}$$

(c) By comparing the magnitude and phase plots for the two systems, determine the effect of having a zero in the right half s-plane rather than the left s-plane. Comment on the terminology "non-minimum phase".

(d) Use MATLAB to plot the step-response (on the same plot) of the two systems

$$H_1(s) = \frac{s+3}{s+1}, \qquad H_2(s) = -\frac{s-3}{s+1}$$

Comment on the initial response (for small t), and the final value.

(d) Consider a system with transfer function $H_2(s)$, with $b = a$. Compute the magnitude and phase responses, and discuss why this might be called an *all-pass filter*.

Problem 5: Back to the lab project (for the final time :-)).
The actions of a tuned-mass damper can be understood from its effect on the frequency response of the damped and undamped systems. In Problem Set 7, Prob. 1 you were given the transfer function for the undamped building velocity v_{m_1} in response to the wind force, and asked to develop the same transfer function for the passively damped building. (See the published solution or the Lab Handout for the transfer functions.)

(a) With the values of the parameters you found in the lab, use MATLAB to make Bode magnitude and phase plots of the frequency response of the undamped and damped buildings.

(b) Use MATLAB to plot a pole/zero plot of both systems.

(c) Explain the characteristics (asymptotic behavior, resonant peaks, etc) of the magnitude and phase plots in terms of the poles and zeros.

(d) Comment on how you would interpret the results to infer the degree of building sway reduction by the passive damper.

(e) From the magnitude plot estimate the factor by which the building sway has been reduced by the addition of the passive components.