2.25 ADVANCED FLUID MECHANICS

QUIZ 1
TUESDAY, OCTOBER 6, 2003, 7:00-9:00 P.M.

OPEN QUIZ WHEN TOLD AT 7:00 PM

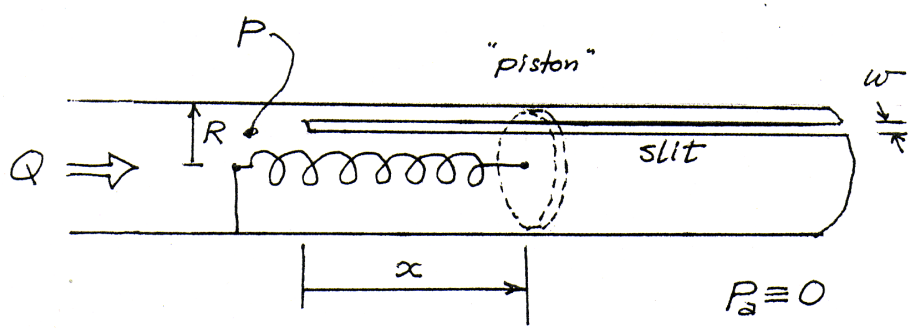
THERE ARE TWO PROBLEMS

2.25 Advanced Fluid Mechanics
Quiz 1, 2003

PROBLEM 1

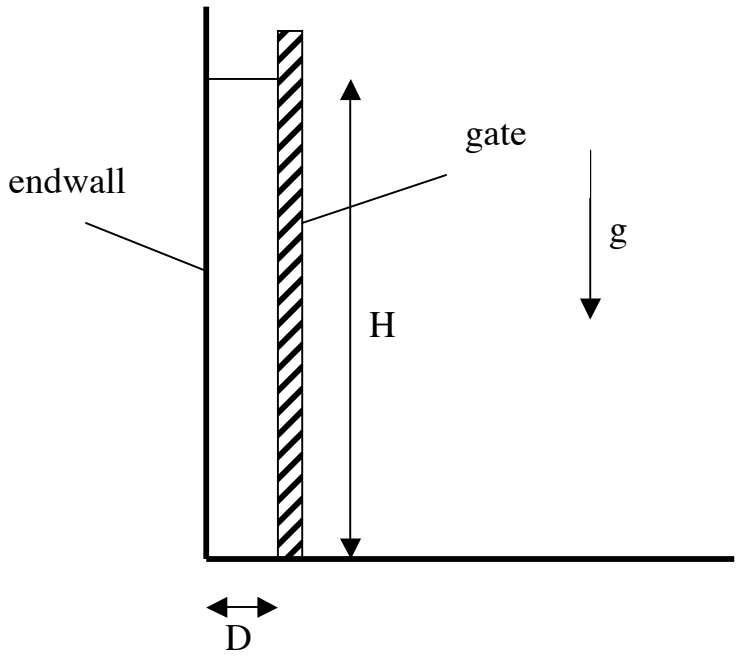
An inventor has proposed an inexpensive, expendable device for measuring lung function in humans. It consists of a round Plexiglas tube (inside radius R) with a slit of width w running down the length of it, as shown below. Inside is a lightweight, freely movable but well-fitting "piston" attached to a linear spring. The equilibrium position of the piston is at x=0, where the slit begins. The piston's equilibrium displacement x is related linearly to the force that is acting on it, $F=kx$, where k is given.

The idea is to have the patient exhale as hard as he can into the tube for a few seconds and measure the piston position x during this time.



Derive equations for (a) the volume flow rate Q, and (b) the gage pressure p in the tube, during the exhalation, expressed in terms of the piston position x and quantities that are either explicitly given above, or that you feel should be given. Assume that the slit width w is so small that even at full piston extension, the outflow area presented by the slit is small compared with the tube's cross-sectional area πR^2.

PROBLEM II



The figure shows a schematic diagram of a narrow deep trench used to hold a heavy fluid. The trench has two vertical sidewalls, separated by a horizontal distance W (in the direction perpendicular to the paper). A gate, which can slide along several rails attached to the sidewalls while remaining vertical and parallel to the endwall, is held in place using several stops fastened to the sidewalls. Due to an unfortunate material fatigue at a moment when the fluid height inside the trench is Ho, the stops break and the gate starts to accelerate towards the right, moving horizontally while remaining vertical and parallel to the endwalls. At the moment when the stops break, the gate is a distance Do away from the endwall of the trench. We are interested in the motion of the gate between Do and D, where D(t) is the instantaneous distance between the gate and the endwall. During this period, we may assume that the following is true: $H >> D$. Moreover, you may assume that the free surface of the fluid remains horizontal during this period. The mass of the gate is M and the density of the fluid is ρ. You may neglect friction between the gate and the ground or the sidewalls.

Derive a differential equation which can be solved in D(t), for D>Do.
You do not have to solve this equation.