Admin

New topic—sampling.

MST

Review Background

- kruskal

- boruvka

- verification

Idea: sample representative subproblem.
Intuition: "fences" like selection algorithm.
sampling theorem:

- Heavy edges

- pick F with probability p

- get n/p F-heavy edges

Recursive algorithm without boruvka:

$$T(m,n) = T(m/2,n) + O(m) + T(2n,n) = O(m + n\log n)$$

(sloppy on expectation on T(2n,n))
Recursive algorithm with 3 boruvka steps:

$$\begin{aligned}
T(m,n) &= T(m/2,n/8) + c_1(m+n) + T(n/4,n/8) \\
&\leq c(m/2 + n/8) + c_1(m+n) + c(n/4 + n/8) \\
&= (c/2 + c_1)m + (c/8 + c_1 + c/4 + c/8)n \\
&= (c/2 + c_1)(m + n)
\end{aligned}$$

so set $c = 2c_1$ (not sloppy expectation thanks to linearity).
Notes:

- Chazelle $m\log\alpha(m,n)$ via relaxed heap

- Ramachandran and Peti optimal deterministic algorithm (runtime unknown)

- open questions.