6.453 Quantum Optical Communication
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6.453 *Quantum Optical Communication* Lecture 20

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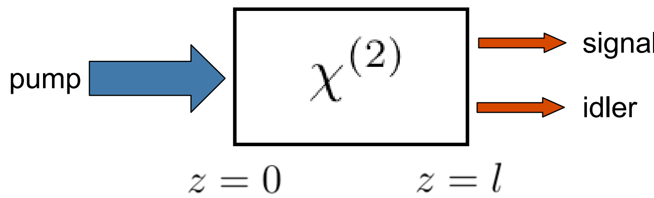
- Announcements
  - Pick up lecture notes, slides
- Nonlinear Optics of $\chi^{(2)}$ Interactions
  - Maxwell's equations with a nonlinear polarization
  - Coupled-mode equations for parametric downconversion
  - Phase-matching for efficient interactions
  - Classical solutions

Second-Order Nonlinear Optics

- Spontaneous Parametric Downconversion



  - Strong pump at frequency $\omega_P = \omega_S + \omega_I$
  - No input at signal frequency ω_S
  - No input at idler frequency ω_I
  - Nonlinear mixing in $\chi^{(2)}$ crystal produces signal and idler outputs

Classical Electromagnetics in Nonlinear Medium

- Maxwell's Equations in a Dielectric Medium:

$$\nabla \times \vec{E}(\vec{r},t) = -\mu_0 \tfrac{\partial}{\partial t}\vec{H}(\vec{r},t), \qquad \nabla \cdot \vec{D}(\vec{r},t) = 0$$

$$\nabla \times \vec{H}(\vec{r},t) = \tfrac{\partial}{\partial t}\vec{D}(\vec{r},t), \qquad \nabla \cdot \mu_0 \vec{H}(\vec{r},t) = 0$$

- Constitutive Relation: $\vec{D}(\vec{r},t) = \epsilon_0 \vec{E}(\vec{r},t) + \vec{P}(\vec{r},t)$

- Wave Equation for $+z$-going Plane Waves:

$$\frac{\partial^2}{\partial z^2}\vec{E}(z,t) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\vec{E}(z,t) - \mu_0 \frac{\partial^2}{\partial t^2}\vec{P}(z,t) = \vec{0}$$

Pump, Signal, and Idler Plane-Wave Modes

- Assume Monochromatic Pump, Signal, and Idler:

$$
\begin{aligned}
\vec{E}(z,t) &= (A_S(z)e^{-j(\omega_S t - k_S z)} + \text{cc})\vec{i}_S/2 \\
&+ (A_I(z)e^{-j(\omega_I t - k_I z)} + \text{cc})\vec{i}_I/2 \\
&+ (A_P e^{-j(\omega_P t - k_P z)} + \text{cc})\vec{i}_P/2
\end{aligned}
$$

  - Non-depleting pump
  - Slowly-varying signal and idler complex amplitudes

Linear and Nonlinear Polarization Terms

- Constitutive Law for Second-Order Nonlinear Crystal:

$$
\begin{aligned}
&\epsilon_0\vec{E}(z,t) + \vec{P}(z,t) \\
&\approx (\epsilon_0 n_S^2(\omega_S)A_S(z)e^{-j(\omega_S t - k_S z)} + \text{cc})\vec{i}_S/2 \\
&+ (\epsilon_0 n_I^2(\omega_I)A_I(z)e^{-j(\omega_I t - k_I z)} + \text{cc})\vec{i}_I/2 \\
&+ (\epsilon_0 n_P^2(\omega_P)A_P e^{-j(\omega_P t - k_P z)} + \text{cc})\vec{i}_P/2 \\
&+ (\epsilon_0 \chi^{(2)} A_I^*(z)A_P e^{-j[(\omega_P - \omega_I)t - (k_P - k_I)z]} + \text{cc})\vec{i}_S/2 \\
&+ (\epsilon_0 \chi^{(2)} A_S^*(z)A_P e^{-j[(\omega_P - \omega_S)t - (k_P - k_S)z]} + \text{cc})\vec{i}_I/2
\end{aligned}
$$

Coupled-Mode Equations for Downconversion

- Photon Fission: $\omega_P = \omega_S + \omega_I$
- Signal and Idler Equations for $0 \leq z \leq l$:

$$\begin{aligned} \frac{\partial A_S(z)}{\partial z} &= j\frac{\omega_S \chi^{(2)}}{2cn_S} A_P A_I^*(z) e^{j(k_P - k_S - k_I)z} \\ \frac{\partial A_I(z)}{\partial z} &= j\frac{\omega_I \chi^{(2)}}{2cn_I} A_P A_S^*(z) e^{j(k_P - k_S - k_I)z} \end{aligned}$$

Conversion to Photon-Units Fields

- Time-Average Powers on Photodetector Active Area $\mathcal{A}$:

$$S_m(z) = \frac{c\epsilon_0 n_m \mathcal{A}}{2} |A_m(z)|^2, \quad \text{for } m = S, I, P$$

- Photon-Units Fields:

$$S_m(z) = \hbar\omega_m |A_m(z)|^2, \quad \text{for } m = S, I, P$$

- Photon-Units Coupled-Mode Equations:

$$\begin{aligned} \frac{\partial A_S(z)}{\partial z} &= j\kappa A_I^*(z) e^{j\Delta k z} \\ \frac{\partial A_I(z)}{\partial z} &= j\kappa A_S^*(z) e^{j\Delta k z} \end{aligned}$$

Type-II Phase Matched Operation at Degeneracy

- Phase Matching for Efficient Coupling: $\Delta k = 0$
  - Conservation of photon momentum: $k_P = k_S + k_I$
  - Type-II system: $\vec{i}_S = \vec{i}_x,\ \vec{i}_I = \vec{i}_y$
- Operation at Frequency Degeneracy: $\omega_S = \omega_I = \omega_P/2$
- Classical Input-Output Relation:

$$A_S(l) = \cosh(|\kappa|l)A_S(0) + j\frac{\kappa}{|\kappa|}\sinh(|\kappa|l)A_I^*(0)$$

$$A_I(l) = \cosh(|\kappa|l)A_I(0) + j\frac{\kappa}{|\kappa|}\sinh(|\kappa|l)A_S^*(0)$$

Coming Attractions: Lectures 21and 22

- Lecture 21:
  Nonlinear Optics of $\chi^{(2)}$ Interactions
  - Quantum coupled-mode equations for parametric downconversion
  - Two-mode Bogoliubov relation
  - Gaussian-state characterization

  Quantum Signatures from Parametric Interactions
  - Squeezed states from parametric amplifiers
- Lecture 22:
  Quantum Signatures from Parametric Interactions
  - Photon twins from parametric amplifiers
  - Hong-Ou-Mandel dip produced by parametric downconversion
  - Polarization entanglement produced by parametric downconversion