6.453 Quantum Optical Communication
Spring 2009

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6.453 *Quantum Optical Communication* Lecture 15

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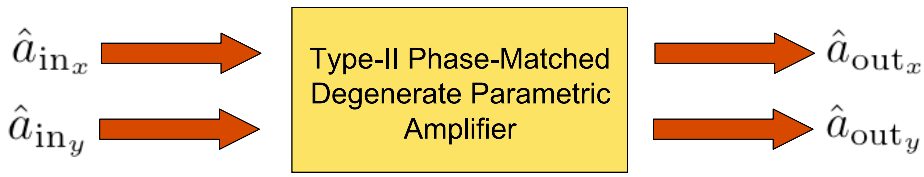
6.453 *Quantum Optical Communication* - Lecture 15

- Announcements
  - Pick up lecture notes, slides
- Teleportation
  - Fidelity analysis of continuous-variable teleportation

Quadrature Entanglement: Vacuum-Input Paramp



- Quadrature Variances:

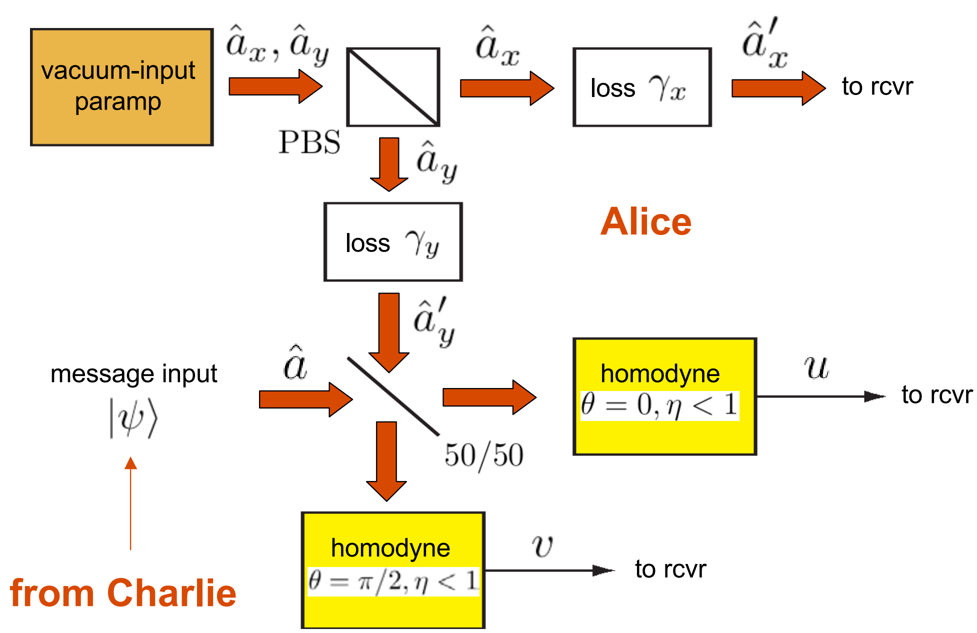
$$\langle \Delta\hat{a}^2_{\text{out}_{x_k}} \rangle = \langle \Delta\hat{a}^2_{\text{out}_{y_k}} \rangle = \frac{2G-1}{4} > \frac{1}{4}, \text{ for } k = 1, 2$$

- Quadrature-Difference Variances:

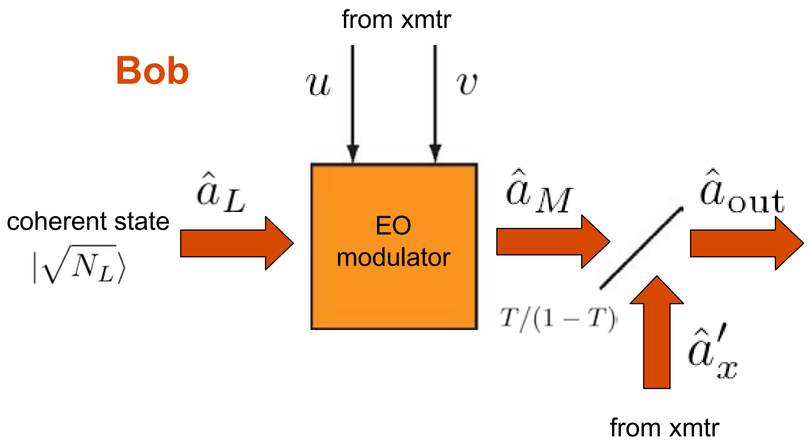
$$\left\langle \left(\frac{\Delta\hat{a}_{\text{out}_{x_1}} - \Delta\hat{a}_{\text{out}_{y_1}}}{\sqrt{2}} \right)^2 \right\rangle = \left\langle \left(\frac{\Delta\hat{a}_{\text{out}_{x_2}} + \Delta\hat{a}_{\text{out}_{y_2}}}{\sqrt{2}} \right)^2 \right\rangle$$

$$= \frac{(\sqrt{G} - \sqrt{G-1})^2}{4} \approx \frac{1}{16G} \ll \frac{1}{4}, \text{ for } G \gg 1$$

Continuous-Variable Teleportation: the Xmtr



Continuous-Variable Teleportation: the Rcvr



Continuous-Variable Teleportation: Xmtr Details

- Inputs to the 50/50 Beam Splitter:

$$\hat{a} \text{ and } \hat{a}'_y = \sqrt{\gamma_y}\,\hat{a}_y + \sqrt{1-\gamma_y}\,\hat{a}_{\gamma_y}$$

where $\hat{a}$ is in arbitrary state, $\hat{a}_{\gamma_y}$ is in vacuum state

- Homodyne Detector Measurement Outcomes:

$$u \quad \leftrightarrow \quad \hat{u} \equiv \sqrt{\eta}\,(\hat{a}_1 + \hat{a}'_{y_1}) + \sqrt{2(1-\eta)}\,\hat{a}_{u_1}$$

$$v \quad \leftrightarrow \quad \hat{v} \equiv \sqrt{\eta}\,(\hat{a}_2 - \hat{a}'_{y_2}) + \sqrt{2(1-\eta)}\,\hat{a}_{v_2}$$

where $\hat{a}_u$ and $\hat{a}_v$ are in vacuum states

Continuous-Variable Teleportation: Xmtr Details

- Homodyne-Measurement Signal-to-Noise Ratios

$$\mathrm{SNR}_u \equiv \frac{\eta\langle \hat{a}_1^2\rangle}{\eta\langle \hat{a}_{y_1}'^2\rangle + 2(1-\eta)\langle \hat{a}_{u_1}^2\rangle}$$

$$= \frac{4\eta\langle \hat{a}_1^2\rangle}{\eta[1+2\gamma_y(G-1)] + 2(1-\eta)}$$

$$\mathrm{SNR}_v \equiv \frac{\eta\langle \hat{a}_2^2\rangle}{\eta\langle \hat{a}_{y_2}'^2\rangle + 2(1-\eta)\langle \hat{a}_{v_2}^2\rangle}$$

$$= \frac{4\eta\langle \hat{a}_2^2\rangle}{\eta[1+2\gamma_y(G-1)] + 2(1-\eta)}$$

Classical Information as Quantum Information

- Alice's Classical Transmission: $u + jv$

- Quantum-Mechanical Equivalent:

$$\hat{u} + j\hat{v} = \sqrt{\eta}\,(\hat{a} + \hat{a}_y'^{\dagger}) + \sqrt{2(1-\eta)}\,(\hat{a}_{u_1} + j\hat{a}_{v_2})$$

$$\leftrightarrow \sqrt{\eta}\,(\hat{a} + \hat{a}_y'^{\dagger}) + \sqrt{1-\eta}\,(\hat{a}_{\eta_u} + \hat{a}_{\eta_v}^{\dagger})$$

where $\hat{a}_{\eta_u}$ and $\hat{a}_{\eta_v}$ are in vacuum states

Continuous-Variable Teleportation: Rcvr Details

- Input to the EO Modulator: Strong coherent state $|\sqrt{N_L}\rangle$
- Output from the EO Modulator: Coherent state $|K(u+jv)\rangle$
- Asymmetric Beam Splitter Input-Output Relation:

$$
\begin{aligned}
\hat{a}_{\text{out}} &= \sqrt{T}\,\hat{a}_M - \sqrt{1-T}\,\hat{a}'_x \\
&= \sqrt{T}\,K(u+jv) + \sqrt{T}\,\Delta\hat{a}_M \\
&- \sqrt{(1-T)\gamma_x}\,\hat{a}_x - \sqrt{(1-T)(1-\gamma_x)}\,\hat{a}_{\gamma_x}
\end{aligned}
$$

where $\Delta\hat{a}_M$ and $\hat{a}_{\gamma_x}$ are in vacuum states

Continuous-Variable Teleportation: Rcvr Details

- Parameter Choices:

$$K\sqrt{\eta T} = 1 \text{ and } \gamma = \gamma_y = (1-T)\gamma_x$$

- Quantum Input-Output Teleportation Relation:

$$
\begin{aligned}
\hat{a}_{\text{out}} &= \hat{a} + \sqrt{\frac{1-\eta}{\eta}}\,(\hat{a}_{\eta_u} + \hat{a}^\dagger_{\eta_v}) \\
&+ \sqrt{1-\gamma}\,(\hat{a}_N + \hat{a}^\dagger_{\gamma_y}) - \sqrt{\gamma}(\hat{a}_x - \hat{a}^\dagger_y)
\end{aligned}
$$

where $\hat{a}_N$ is in the vacuum state

Continuous-Variable Teleportation: Output State

- Wigner Characteristic Function of Output State:

$$\chi_W^{\rho_{\text{out}}}(\zeta^*, \zeta) = \chi_W^{\rho}(\zeta^*, \zeta) \times \exp\left\{-\left[\left(\frac{1-\eta}{\eta}\right) + (1-\gamma) + \gamma s\right]|\zeta|^2\right\}$$

where $s \equiv (\sqrt{G} - \sqrt{G-1})^2$ is the squeezing level

Output State Fidelity

- Fidelity Analysis:

$$F \equiv \langle\psi|\hat{\rho}_{\text{out}}|\psi\rangle = \langle\psi|\int \frac{d^2\zeta}{\pi}\,\chi_A^{\rho_{\text{out}}}(\zeta^*, \zeta)e^{-\zeta\hat{a}_{\text{out}}^\dagger}e^{\zeta^*\hat{a}_{\text{out}}}|\psi\rangle$$

- Coherent-State Input Special Case:

$$F = \frac{1}{1 + \left(\frac{1-\eta}{\eta}\right) + (1-\gamma) + \gamma s}$$

Coming Attractions: Lecture 16 + Schedule Notes

- Lecture 16:
  Quantum Cryptography
  - Bennett-Brassard protocol
  - Ekert protocol

- Schedule Notes:
  - Term paper proposals are due Thursday, November 6
  - Mid-term exam Thursday, November 6