6.453 Quantum Optical Communication
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6.453 *Quantum Optical Communication* Lecture 10

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6.453 *Quantum Optical Communication* - Lecture 10

- Announcements
  - Turn in problem set 5
  - Pick up problem set 4 graded, problem set 5 solution, problem set 6, lecture notes, slides
- Single-Mode Photodetection
  - Signatures of non-classical light
  - Squeezed-state waveguide tap

Single-Mode Semiclassical Photodetection

- Photon-Units Classical Field:

$$E(t) = \frac{ae^{-j\omega t}}{\sqrt{T}}, \quad \text{for } 0 \le t \le T$$

- Direct Detection: given a , N is Poisson with mean $|a|^2$
- Homodyne Detection: given a , $\alpha_\theta \sim N(a_\theta, 1/4)$
- Heterodyne Detection: given a ,

$$\{\alpha_1, \alpha_2\} \text{ SI}, \ \alpha_i \sim N(a_i, 1/2)$$

Single-Mode Quantum Photodetection

- Photon-Units Field Operator:

$$\hat{E}(t) = \underbrace{\frac{\hat{a}e^{-j\omega t}}{\sqrt{T}}}_{\text{excited mode}} + \underbrace{\text{other terms}}_{\text{unexcited modes}}$$

for $0 \le t \le T$

- Direct Detection: $\hat{N} = \hat{a}^\dagger \hat{a}$ measurement
- Homodyne Detection: $\hat{a}_\theta = \text{Re}(\hat{a}e^{-j\theta})$ measurement
- Heterodyne Detection: $\hat{a}$ measurement

Single-Mode Random Fields: Classical vs. Quantum

- Classical Field: $a = a_1 + ja_2$ complex random variable
  - Measurement variances

Direct Detect. N	Homodyne Det. α_1	Heterodyne Det. α_1
$\langle \lvert a\rvert^2\rangle + \langle \Delta(\lvert a\rvert^2)^2\rangle$	$\frac{1}{4} + \langle \Delta a_1^2\rangle$	$\frac{1}{2} + \langle \Delta a_1^2\rangle$

- Quantum Field: $\hat{\rho}_a$ density operator of the excited mode
  - Measurement variances

Direct Detect. $\hat{N}$	Homodyne Det. $\hat{a}_1$	Heterodyne Det. $\hat{a}_1 + \hat{a}_{I_1}$
$\langle \Delta \hat{N}^2\rangle$	$\langle \Delta \hat{a}_1^2\rangle$	$\langle \Delta \hat{a}_1^2\rangle + \frac{1}{4}$

where $\langle \hat{A}\rangle = \mathrm{tr}(\hat{\rho}_a \hat{A})$

Signatures of Non-Classical Light

- "Classical Light" = random mixture of coherent states

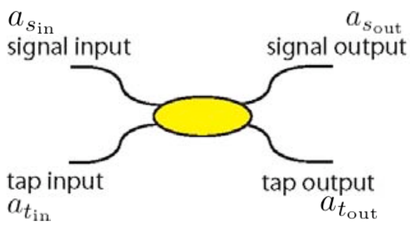
$$\hat{\rho}_a = \int \mathrm{d}^2\alpha\, P(\alpha,\alpha^*)|\alpha\rangle\langle\alpha|, \quad P(\alpha,\alpha^*) \text{ a 2-D pdf}$$

- Sub-Poissonian Statistics: $\langle \Delta\hat{N}^2\rangle < \langle \hat{N}\rangle$
- Quadrature-Noise Squeezing: $\langle \Delta\hat{a}_\theta^2\rangle < \dfrac{1}{4}$
- Heterodyne-Detection Statistics Determine $\hat{\rho}_a$

$$\hat{\rho}_a = \int \frac{\mathrm{d}^2\zeta}{\pi}\, \chi_A^{\hat{\rho}_a}(\zeta^*,\zeta) e^{-\zeta\hat{a}^\dagger} e^{\zeta^*\hat{a}}, \quad \chi_A^{\hat{\rho}_a}(\zeta^*,\zeta) \overset{\mathcal{F}}{\longleftrightarrow} \langle\alpha|\hat{\rho}_a|\alpha\rangle$$

Optical Waveguide Tap — Semiclassical

Fused Fiber Coupler



- Coupler is a beam splitter

$$
\begin{aligned}
a_{s_{\text{out}}} &= \sqrt{T}a_{s_{\text{in}}} + \sqrt{1-T}a_{t_{\text{in}}} \\
a_{t_{\text{out}}} &= \sqrt{1-T}a_{s_{\text{in}}} - \sqrt{T}a_{t_{\text{in}}}
\end{aligned}
$$

- Tap input is zero
- Homodyne SNR at signal input

$$\text{SNR}_{\text{in}} = 4|a_{s_{\text{in}}}|^2$$

- Homodyne SNR at signal output

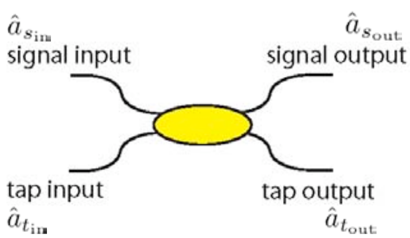
$$\text{SNR}_{\text{out}} = 4T|a_{s_{\text{in}}}|^2$$

- Homodyne SNR at tap output

$$\text{SNR}_{\text{tap}} = 4(1-T)|a_{s_{\text{in}}}|^2$$

Optical Waveguide Tap — Quantum

Fused Fiber Coupler



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\hat{a}_{t_{\text{out}}} &= \sqrt{1-T}\hat{a}_{s_{\text{in}}} - \sqrt{T}\hat{a}_{t_{\text{in}}}
\end{aligned}
$$

- Tap input is in squeezed vacuum
- Homodyne SNR at signal input

$$\text{SNR}_{\text{in}} = 4|a_{s_{\text{in}}}|^2$$

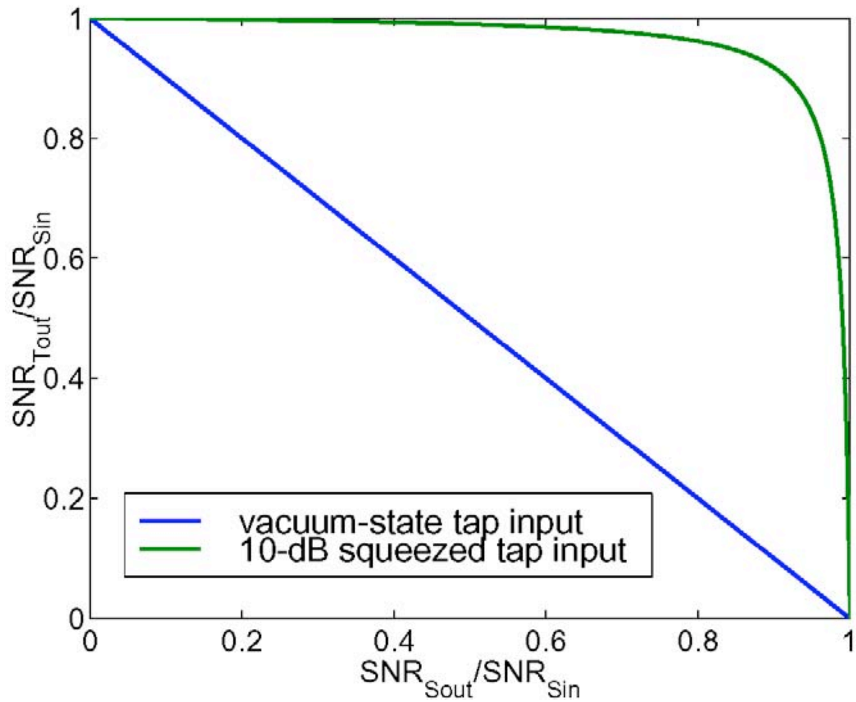
- Homodyne SNR at signal output

$$\text{SNR}_{\text{out}} = \frac{4T|a_{s_{\text{in}}}|^2}{T + (1-T)(\mu-\nu)^2}$$

- Homodyne SNR at tap output

$$\text{SNR}_{\text{tap}} = \frac{4(1-T)|a_{s_{\text{in}}}|^2}{(1-T) + T(\mu-\nu)^2}$$

Optical Waveguide Tap: SNR Tradeoff



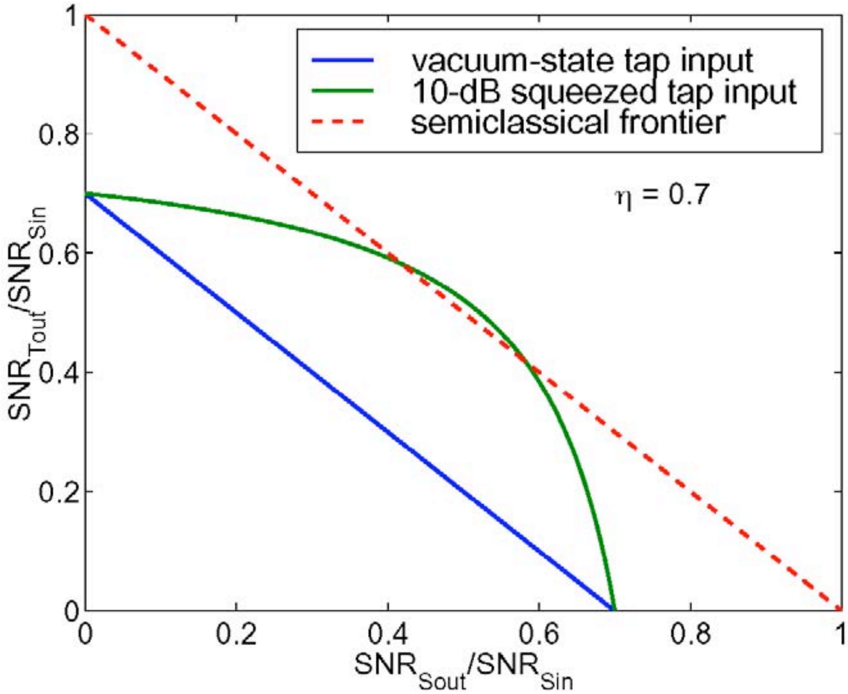
Non-Ideal Quantum Photodetection

- Quantum Efficiency $\eta < 1$:

$$\hat{a}' \equiv \sqrt{\eta}\,\hat{a} + \sqrt{1-\eta}\,\hat{a}_v, \quad \hat{a}_v \text{ in vacuum state}$$

- Direct Detection: $\hat{a}'^\dagger \hat{a}'$ measurement
- Homodyne Detection: $\mathrm{Re}(\hat{a}' e^{-j\theta})$ measurement
- Heterodyne Detection: $\hat{a}'$ measurement

Loss is a Problem for Squeezed States



Coming Attractions: Lecture 11

- Lecture 11:
  Single-Mode and Two-Mode Linear Systems
  - Attenuators
  - Phase-Insensitive Amplifiers
  - Phase-Sensitive Amplifiers