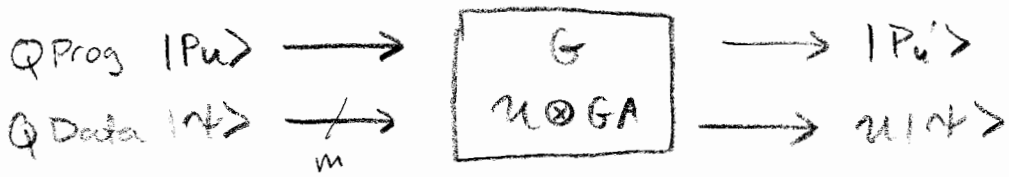
Lecture 9: Teleportation-Based QC

1) Universal qgate array?
2) Teleporting gates
3) Remote gates

① Universal QGate Array



Classical: M bits

~ $2^m!$ possible functions ~ $2^{2^m m}$ (?)

Quantal: M qubits

$\mathcal{U} \sim 2^{2m}$ real #s

- Any $\mathcal{U}$ can be approximated efficiently
- Most $\mathcal{U}$'s are not efficiently describable

2m qubits have ~ $2^{2m}-1$ DOFs

- Use to specify $|P_u\rangle$
- Need: $G\left[|P_u\rangle|\psi\rangle\right] = |P_u'\rangle \otimes \mathcal{U}|\psi\rangle$

Thm: No such deterministic G exists

Pf of Thm: $\nexists$ deterministic G s.t. $G[|P_U\rangle|\psi\rangle] = |P_U'\rangle \otimes U|\psi\rangle$

1) Assume $\exists P, Q$ s.t. U_P & U_Q are unique

$$G[|P\rangle|\psi\rangle] = |P'\rangle \otimes U_P|\psi\rangle$$
$$G[|Q\rangle|\psi\rangle] = |Q'\rangle \otimes U_Q|\psi\rangle$$

$$\langle\psi|\psi\rangle\langle P|Q\rangle = \langle P'|Q'\rangle\langle\psi|U_P^\dagger U_Q|\psi\rangle$$

$$\frac{\langle P|Q\rangle}{\langle P'|Q'\rangle} = \langle\psi|U_P^\dagger U_Q|\psi\rangle$$

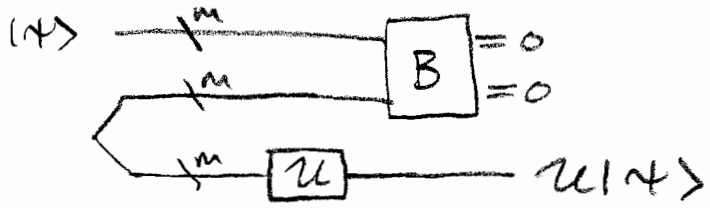
indep of $|\psi\rangle$ $\Rightarrow$ Must be indep of $|\psi\rangle$

So $U_P^\dagger U_Q = \gamma I$ (γ = some const)

✗ since then P, Q nondistinct

Thus, every implementable U requires an extra Hilbert space dimension in the program register. □

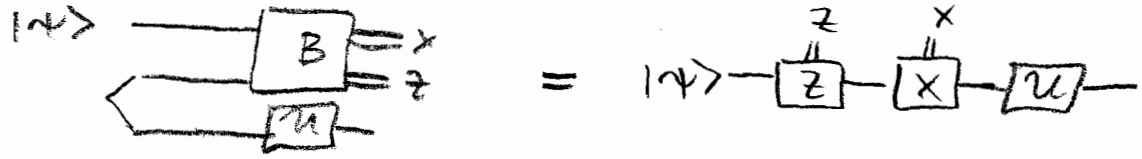
Probabilistic Version



... with prob 2^{-2m}

② Teleporting Gates

Claim:

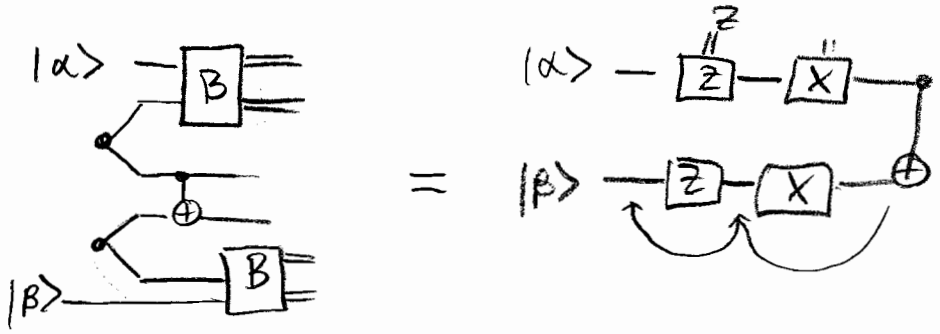


Fact:

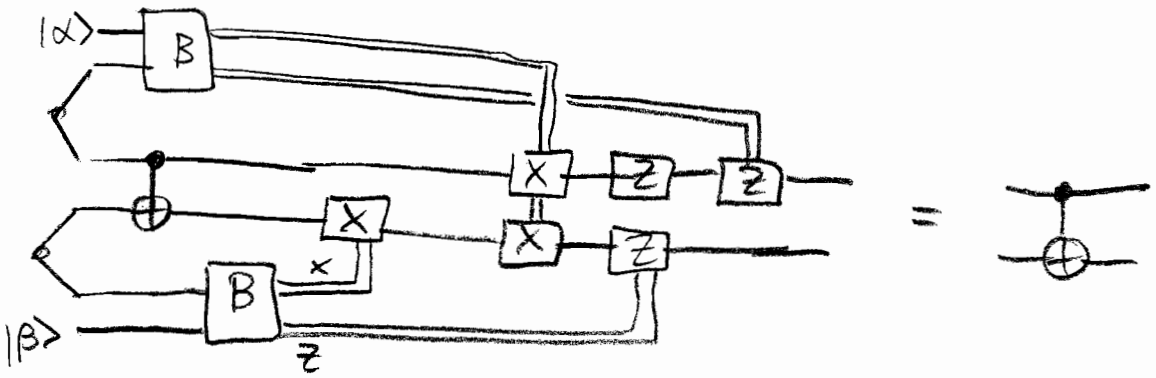
Ex: H:

Push H up to front by commuting others (?)
Can fix up ... ?

Ex: CNOT:

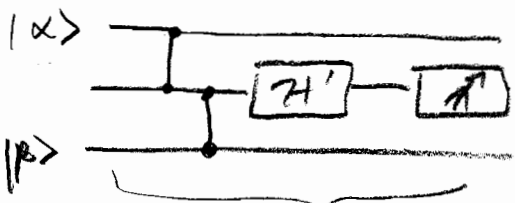


Thus,



③ Remote Gates

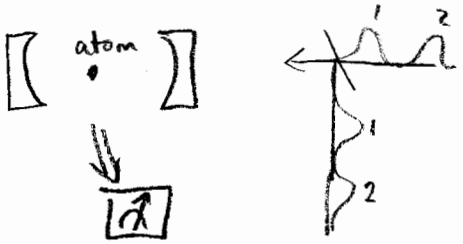
Remote CNOT:



$H' := \frac{1}{\sqrt{2}}(Z+Y)$

Def: $R_d := R_z\left((-1)^{d+1}\,\pi/2\right)$ $\rightarrow (R_d \otimes R_d)\,\underbrace{\wedge(Z)}_{\text{CPHASE}}$

eg. [[atom]]



Measure atom in such a way as to make cphase gate