MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Departments of Electrical Engineering, Mechanical Engineering, and the Harvard-MIT Division of Health Sciences and Technology

6.022J/2.792J/BEH.371J/HST542J: Quantitative Physiology: Organ Transport Systems

PROBLEM SET 5

Assigned: March 9, 2004

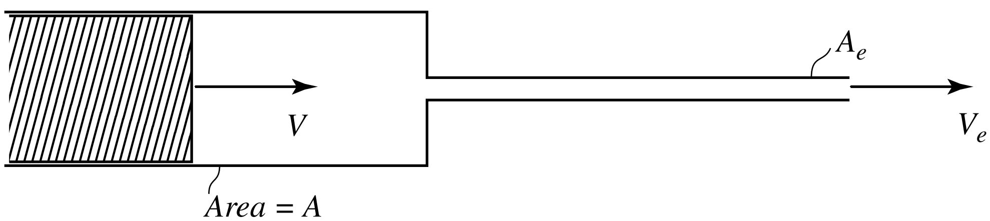
Due: March 16, 2004

Problem 1

A. The hypodermic needle in the figure below contains a saline solution. If a plunger of area A is pushed in at a steady rate (V), what is the mean exit velocity (V_e) of solution leaving the needle of area A_e? Assume no leakage past the plunger.

B. If there is leakage back past the plunger equal to one-third the volume flow rate from the needle, find an expression for V_e.

C. Neglecting leakage past the plunger, find an expression for the pressure at the face of the plunger if the fluid exits the needle at atmospheric pressure *and* the fluid can be treated as though it were inviscid. The flow can be treated as steady.

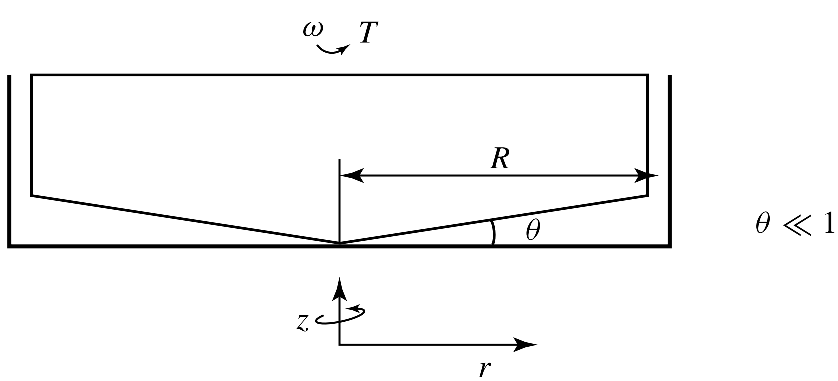


2004/241

Problem 2

A common type of viscometer consists of a cone rotating against a fixed plate, as shown in Figure 1. Show from physical arguments (or otherwise) that the shear rate is independent of r. (*Hint:* $v_\phi = A(r)z$.) Explain how this viscometer can be used to construct the stress-strain relationship of a non-Newtonian fluid like blood, when the torque T on the cone and the angular speed ω are known.

Figure 1:



2004/43

Problem 3

Consider laminar viscous flow in a cylindrical vessel. Show that the magnitude of the shear rate at the wall is given by:

$$\dot{\gamma} = \left.\frac{\partial v}{\partial r}\right|_{\text{wall}} = \frac{8\bar{v}}{D}$$

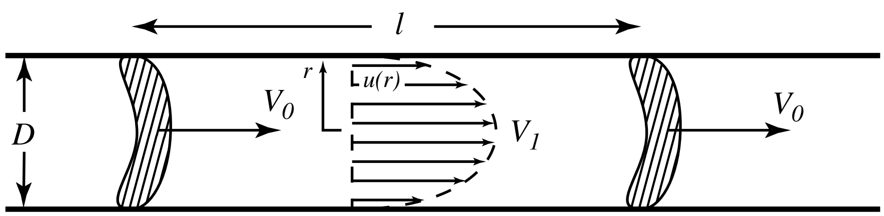
where $\bar{v}$ is the average flow velocity through the vessel and D is the diameter.

2004/14

Problem 4

One simple and instructive model of the flow of erythrocytes through the capillaries is shown in the sketch below. The erythrocyte fills the tube so that a bolus of plasma is trapped between each pair of cells and travels with the cells.

If the distance between cells, l, is large compared to the capillary diameter D, the velocity profile in the plasma between the cells is nearly that of a Poiseuille flow. Show that the plasma centerline velocity, V_1, is twice the erythrocyte velocity V_0.



2004/21

Problem 5

A. A patient has a diseased aortic valve. The valve does not leak, but it has stenosis leading to maximum velocity of 5 meter/sec exiting the valve. If the peak flow rate in systole through the valve is 350 ml/sec and the left ventricular outflow tract area is 3.2 square centimeters, what is the maximal systolic gradient across the valve? What are the assumptions that you made, and why are they reasonable?

B. Occasional patients have stenosis of the aortic valve, but also have a narrowed left ventricular outflow tract just proximal to the valve. If the maximal systolic velocity exiting the aortic valve stenosis is 5 meters/sec and the flow rate is 350 ml/sec but the outflow tract area is now 1.5 square centimeters, what is the maximal gradient across the valve?

C. For both of the above cases, calculate the area of the vena contracta (the area of the smallest region of the jet). Is the TRUE valve area larger or smaller than the vena contracta area?

2004/558