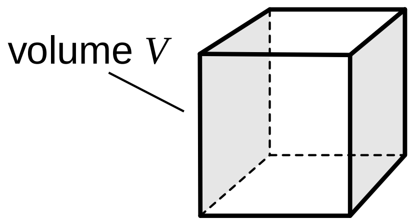
Harvard-MIT Division of Health Sciences and Technology
HST.410J: Projects in Microscale Engineering for the Life Sciences, Spring 2007
Course Directors: Prof. Dennis Freeman, Prof. Martha Gray, and Prof. Alexander Aranyosi

HST.410J/6.07J **Lecture 5**

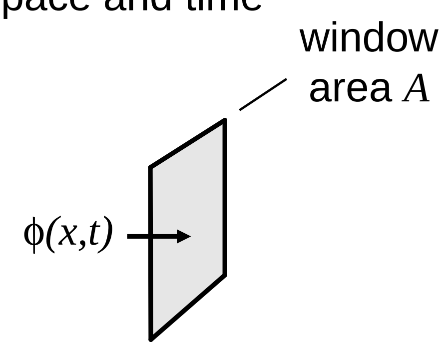
February 22, 2007

Concentration at a point in space and time



$$\text{concentration } c(x,t) = \lim_{V \to 0} \frac{\text{amount of substance in } V}{V}$$

Flux at a point in space and time



$$\text{flux } \phi(x,t) = \lim_{\substack{A \to 0 \\ \Delta t \to 0}} \frac{\text{amount of substance flowing through test window } A \text{ in } \Delta t}{A \Delta t}$$

Fick's First Law

$$\phi(x,t) = -D \frac{\partial c(x,t)}{\partial x}$$

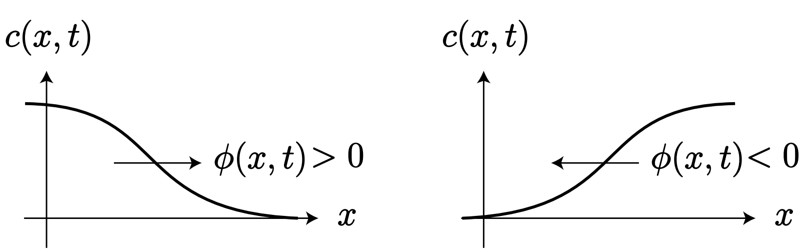
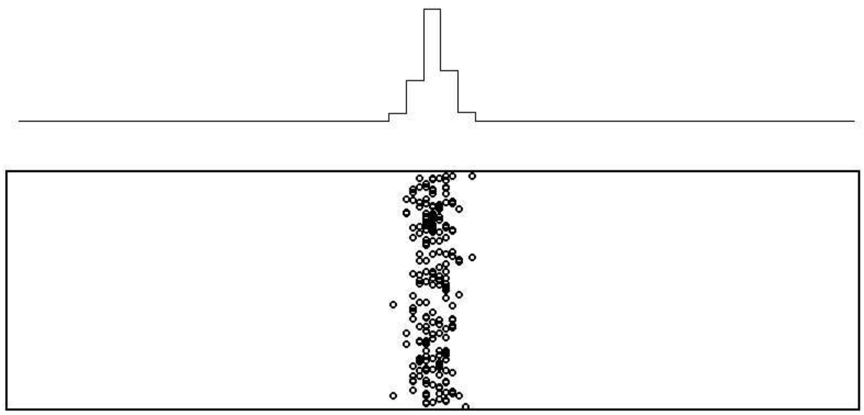
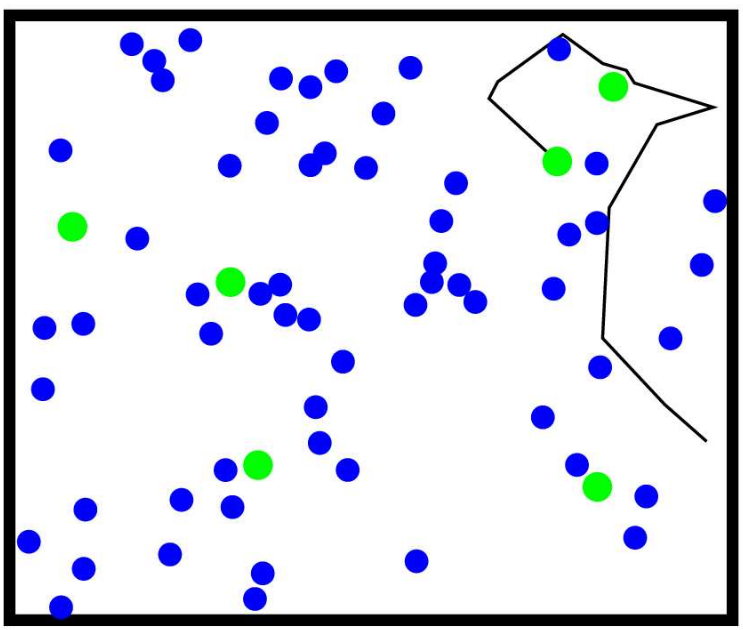
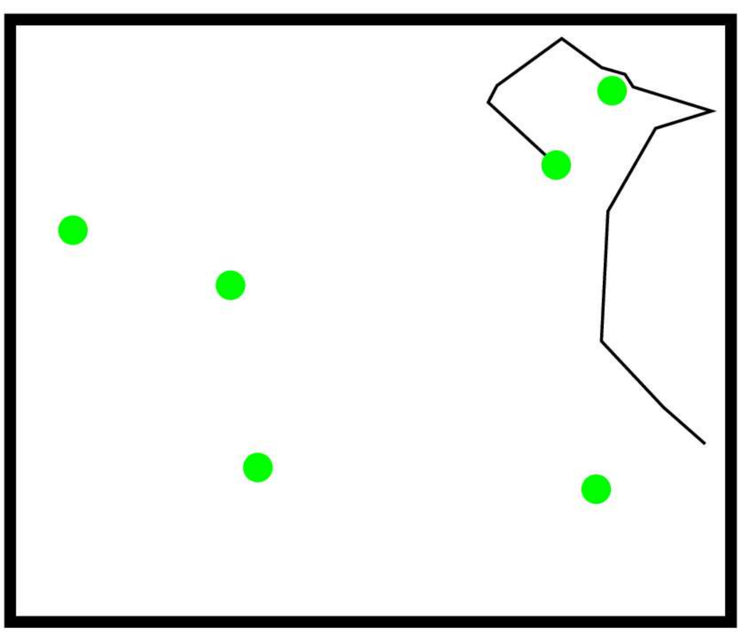


Figure from Weiss, T. F. *Cellular Biophysics, Vol. I.* Cambridge, MA: MIT Press, 1996. Courtesy of MIT Press. Used with permission.





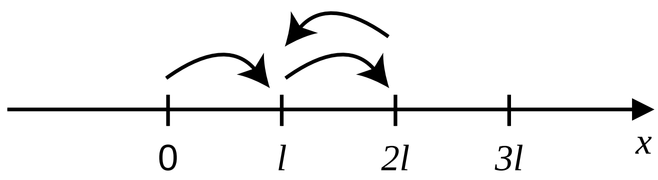


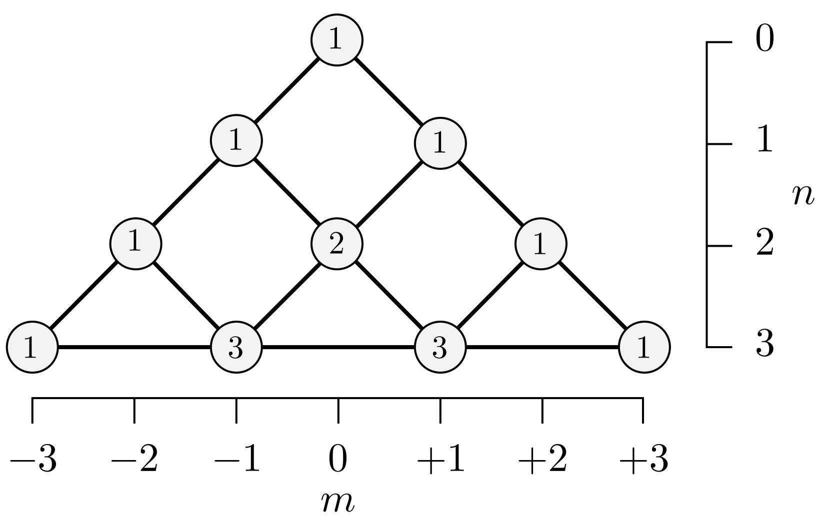
Random Walk Model

- number of solute particles << number of solvent particles
- motion of solute determined by collisions with solvent (ignore solute-solute interactions)
- focus on 1 solute particle, assume motions of others are statistically identical

Every τ seconds, solute particle gets hit by solvent particle.

In response, solute particle is equally likely to move $+l$ or $-l$.
τ = mean free time; l = mean free path





Figures from Weiss, T. F. *Cellular Biophysics, Vol. I.* Cambridge, MA: MIT Press, 1996.
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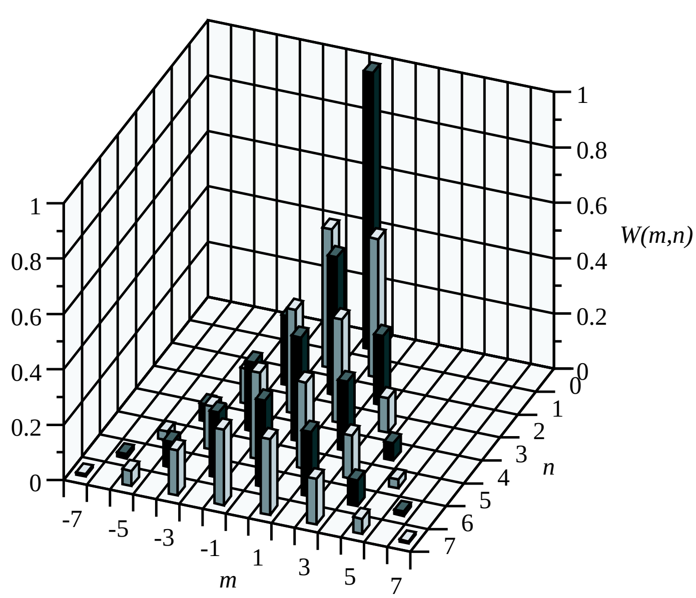

Fick's First Law

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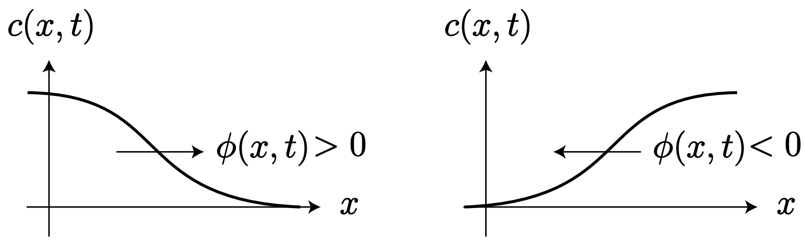

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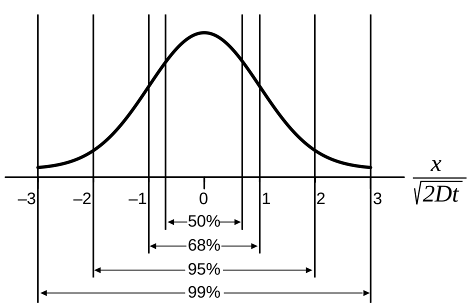
How long till half the solute diffuses to $|x| > x_{1/2}$



$$c(x,t) = \frac{n_o}{\sqrt{4\pi Dt}}\, e^{-x^2/(4Dt)}$$

$$\frac{x_{1/2}}{\sqrt{2Dt}} \lesssim \frac{2}{3}$$

$$\frac{2}{3}\sqrt{2Dt} \gtrsim x_{1/2}$$

$$\frac{4}{9}\,2Dt \gtrsim x_{1/2}^2$$

$$t \gtrsim \frac{x_{1/2}^2}{D} \equiv t_{1/2}$$

Importance of Scale

$$t_{1/2} = \frac{x_{1/2}^2}{D} \quad ; \quad D = 10^{-5}\,\frac{\mathrm{cm}^2}{\mathrm{s}} \text{ for small solutes (e.g., Na}^+\text{)}$$

	$x_{1/2}$	$t_{1/2}$
membrane sized	10 nm	$\frac{1}{10}$ μsec
cell sized	10 μm	$\frac{1}{10}$ sec
dime sized	10 mm	10^5 sec ≈ 1 day