

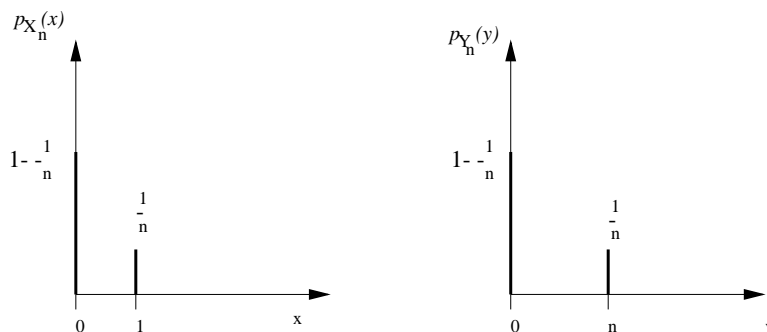
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Spring 2006)

Tutorial 9
April 20-21, 2006

1. **Signal-to-Noise Ratio:** If random variable X has mean $\mu \neq 0$ and standard deviation $\sigma > 0$, the ratio $r = |\mu|/\sigma$ is called the measurement *signal-to-noise ratio* or *SNR*, of X . The idea is that X can be expressed as $X = \mu + (X - \mu)$, with μ representing a deterministic, constant-valued “signal” and $(X - \mu)$ the random, zero-mean “noise.” If we define $|(X - \mu)/\mu| = D$ as the relative deviation of X from its mean μ , show that for $\alpha > 0$,

$$\mathbf{P}(D \leq \alpha) \geq 1 - \frac{1}{r^2 \alpha^2} \quad .$$

2. In your summer internship, you are working for the largest producer of lightbulbs. Your manager asks you to estimate the quality of the production; that is, to estimate p , the probability of a bulb produced by the factory to be defectless. You are told to assume that the quality of each bulb is independent, and identically distributed.
- (a) Supposing you test n randomly picked bulbs, what is a good estimate for p , Z_n , such that Z_n converges to p in probability?
- (b) The management asks that the estimate is located in the range $p \pm 0.1$ with probability 0.95. Are 27 randomly picked bulbs enough for this specification? Give the reason.
- 3.



Let X_n and Y_n have the distributions shown above.

- (a) Evaluate the expectation and variance for X_n and Y_n .
- (b) What does the Chebyshev inequality tell us about the convergence of X_n ?
- (c) What does the Chebyshev inequality tell us about the convergence of Y_n ?
- (d) Is X_n convergent in probability? If so, to what value? Explain.
- (e) Is Y_n convergent in probability? If so, to what value? Explain.
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