MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Spring 2006)

Recitation 4
February 23, 2006

1. **The birthday problem.** Problem 1.45, page 66 in the text.

   Consider n people who are attending a party. What is the probability that each person has a distinct birthday?

   Assume that each person has an equal probability of being born on each day during the year, independently of everyone else, and ignore the additional complication presented by leap years (i.e., nobody is born on February 29).

2. Recall from Lecture 4 the different cases that arise from the problem of selecting/sampling k balls from an urn containing n numbered balls, numbered 1 through n:

   - Sampling with replacement and ordering
   - Sampling without replacement and ordering
   - Sampling without replacement and without ordering
   - Sampling with replacement and without ordering

   The objective of this problem is to study the fourth case. A distinct solution may be expressed in terms of the vector of nonnegative integers $(N_1, N_2, \ldots, N_n)$, where N_i is the number of times the ball numbered i gets selected.

   (a) Explain why we must have $N_1 + N_2 + \cdots + N_n = k$.

   (b) How many distinct solutions does the equation above have? Explain why this is answer to the number of distinct results of sampling with replacement, without ordering.

   (c) Let X_1 denote the number of balls selected that are numbered 1. For any $\ell \in \{0, 1, \ldots, k\}$, find the number of distinct samplings with replacement, without ordering such that $X_1 = \ell$. Use this to state an identity for binomial coefficients.

3. 4 buses carrying 148 job-seeking MIT students arrive at a job fair. The buses carry, respectively, 40, 33, 25, and 50 students. One of the students is randomly selected. Let X denote the number of students that were on the bus carrying this randomly selected student. Also, one of the 4 bus drivers is also randomly selected. Let Y denote the number of students on his bus.

   (a) Do you think $E[X]$ and $E[Y]$ are equal? If not, which is larger? Give your reasoning informally.

   (b) Compute $E[X]$ and $E[Y]$.