Massachusetts Institute of Technology
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Spring 2006)

Problem Set 11:
Topic: Markov Processes
Due: May 12, 2006

1. At the Probability Coffee House of MIT, there is only one cashier. Due to the limited space, she allows only m customers to line before her at any time. If a customer finds there are m customers there including the one being served by the cashier, he will leave the Coffee House immdediately.

   Every minute, exactly one of the following occurs:

   - one new customer arrives with probability p;
   - one existing customer leaves with probality kq, where k is the number of customers in the House; or
   - no new customer arrives and no existing customer leaves with probability $1 - p - kq$ if there is at least one customer in the House, and with probability $1 - p$ otherwise.

   (a) This problem can be modeled as a birth-death process. Define appropriate states and draw the transition probability graph.

   (b) After the House has been open for a long time, you walk into the House. Calculate how many customers you expect to see in line.

2. Sam and Pat are playing foosball. When they begin, the score is 0-0. To make things interesting, if the score ever becomes tied, it is instantly reset to 0-0. Starting from any score, the probability that Sam gets the next point is $\frac{1}{3}$. The game stops when one player's score reaches 2.

   (a) Draw an appropriate Markov chain that describes the game.

   (b) Identify all transient, recurrent, and periodic states.

   (c) What is the probability that Pat wins?