Cytoskeletal mechanics : - readings RDK Ch 2.2 pp. 1-15, 24-54
Ch 2.3 all

Last time : cell membrane mechanics

important concepts : bending modulus K_B, extension modulus K_e, shear modulus K_s
surface tension N

lipid bilayer	$K_B \sim 10^{-19}$ N m^2	K_e high	$K_s \simeq 0$	
with cortex (0.1 μm)	10^{-18}	high	low	N ≈ cst (leukocyte)
				N = 70 dyn / cm (water)

Today - structural elements of cytoskeleton
- scaling & dimensional analysis (see Sonin on the website)
- experimental measurements of cell mechanics (examples using scaling & dimensional analysis)

Later - microstructural models

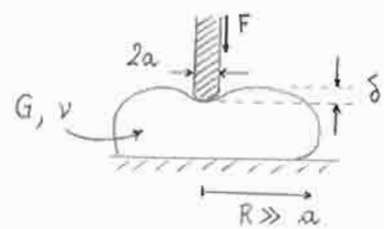
- Structural elements :
  - actin : microfilaments
  - tubulin : microtubules
  - intermediate filaments

  } all important « sometimes »

- Dimensional analysis

1. identify a complete set of independent quantities

indentation



Q_0: δ

$Q_1 \ldots$: $\downarrow$ G, ν, $\cancel{V}$, F, a, $\cancel{R}$

units L $\frac{F}{L^2}$ - F L

2. list the dimensions of Q_i
3. choose a complete & dimensionally independent subset $Q_1, \ldots, Q_k$: G, F, a
4. dimensionless forms of Q_0 : $\Pi_0 = \frac{\delta}{a}$, $\Pi_1 = \nu$, $\Pi_2 = \frac{F}{Ga^2}$
5. rewrite : $\delta = fn\,(G, \nu, F, a)$

$$\frac{\delta}{a} = fn\,\Big(\underbrace{\nu, \frac{F}{Ga^2}}_{\text{dimensionless}}, \underbrace{\cancel{G, a}}_{}\Big)$$

these cannot be made dimensionless ⟹ drop them out

- scaling analysis : useful when (i) exact solutions aren't needed or not warranted
  (ii) present experimental results in generalizable form
  (iii) minimize the number of experiments needed to completely characterize a relationship

- review of governing equations :
  - force balance (neglect inertia) $\frac{\partial \sigma_{ij}}{\partial x_i} = 0$ (for solid & fluid)

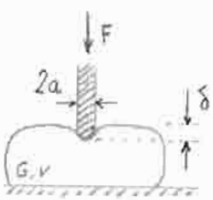
	solid (Hookean)	fluid (Newtonian)
- constitutive equations	$\sigma_{ij} = \lambda \varepsilon_{kk} \delta_{ij} + 2G\varepsilon_{ij}$	$\sigma_{ij} = -p\delta_{ij} + 2\mu \dot{\varepsilon}_{ij}$
	displacement u	velocity v
	strain ε	strain rate $\dot{\varepsilon}$
	stress / strain : (E, ν) or (G, ν)	stress / strain rate (μ, K) K: bulk compression
- energy	$U = \int \frac{1}{2} \underline{\underline{\sigma}} \cdot \underline{\underline{\varepsilon}} \, d\forall$ $\sim E\varepsilon^2 \forall$	$\dot{U} = \int \frac{1}{2} \underline{\underline{\tau}} \cdot \underline{\underline{\dot{\varepsilon}}} \, d\forall$ dissipation rate $\sim \mu \dot{\varepsilon}^2 \forall$

- scaling $\frac{\partial}{\partial x} \sim \frac{1}{L}$, $\frac{\partial}{\partial t} \sim \frac{1}{\tau} (\sim \omega)$, $\int f\,dx \sim fL$, $\int f\,dt \sim f\tau$

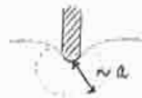
ignore all $O(1)$ dimensionless constants (π, 2 ...)

- Experimental measurements

① cell indentation



scaling force balance $F \sim \tau a^2 \sim G\varepsilon a^2 \sim G\frac{\delta}{a} a^2 \sim G\delta a$



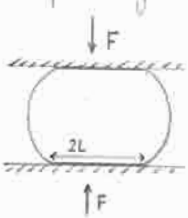
the exact solution would be $F = 8G\delta a$

energy : relate internal energy to work done by external force

$G\varepsilon^2 a^3 \sim G a^3 \left(\frac{\delta}{a}\right)^2 \sim F\delta$

hence $\boxed{F \sim G\delta a}$

cell squashing



$\delta \sim \frac{L^2}{R}$ from

energy $F\delta \sim G\left(\frac{\delta}{L}\right)^2 L^3$

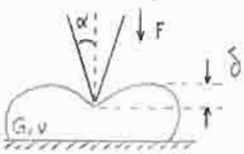
$F \sim G\delta L \sim G\frac{L^3}{R}$ hence $L \sim \left(\frac{FR}{G}\right)^{1/3}$

$\delta \sim \left(\frac{FR}{G}\right)^{2/3} \cdot \frac{1}{R} \sim R^{-1/3} \left(\frac{F}{G}\right)^{2/3}$

$\boxed{F \sim \delta^{3/2} G R^{1/2}}$ Hertz theory

the theory would give us $F = \frac{4}{3} \cdot \frac{E}{1-\nu^2} \delta^{3/2} R^{1/2}$

indentation by AFM probe



dimensional analysis

$F = fcn(\delta, G, \nu, \alpha)$

$\frac{F}{G\delta^2} = fn(\nu, \alpha)$ or $\boxed{F \sim G\delta^2 fcn(\nu, \alpha)}$

exact : $F = \frac{4}{\pi} G\delta^2 \frac{\tan\alpha}{1-\nu}$