Problems for Recitation 22

1 Properties of Variance

In this problem we will study some properties of the variance and the standard deviation of random variables.

a. Show that for any random variable R, $\operatorname{Var}[R] = \operatorname{E}[R^2] - \operatorname{E}^2[R]$.

b. Show that for any random variable R and constants a and b, $\operatorname{Var}[aR + b] = a^2 \operatorname{Var}[R]$. Conclude that the standard deviation of $aR+b$ is a times the standard deviation of R.

c. Show that if R_1 and R_2 are independent random variables, then

$$\operatorname{Var}[R_1 + R_2] = \operatorname{Var}[R_1] + \operatorname{Var}[R_2].$$

d. Give an example of random variables R_1 and R_2 for which

$$\operatorname{Var}[R_1 + R_2] \neq \operatorname{Var}[R_1] + \operatorname{Var}[R_2].$$

e. Compute the variance and standard deviation of the Binomial distribution $H_{n,p}$ with parameters n and p.

f. Let's say we have a random variable T such that $T = \sum_{j=1}^{n} T_j$, where all of the T_j's are mutually independent and take values in the range $[0, 1]$. Prove that Var(T)≤Ex(T). We'll use this result in lecture tomorrow. *Hint: Upper bound* $\operatorname{Var}[T_j]$ *with* $\operatorname{E}[T_j]$ *using the definition of variance in part (a) and the rule for computing the expectation of a function of a random variable.*

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