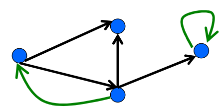
Mathematics for Computer Science

MIT 6.042J/18.062J

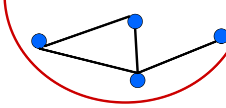
Undirected Graphs: Isomorphism

Types of Graphs

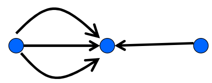
Directed Graphs



Undirected Graphs
(Simple)



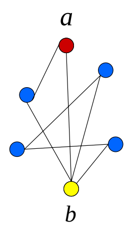
Multi-Graphs



Definition

An Undirected Graph is a set of vertices V and a set of edges E where each edge is an unordered pair of distinct vertices a and b.

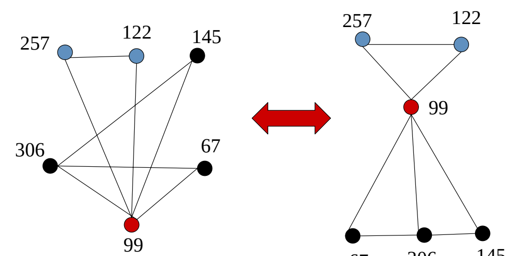
a—b (edge ab) = $\{a,b\}$



Degree of a vertex v is the number of edges it connects to.

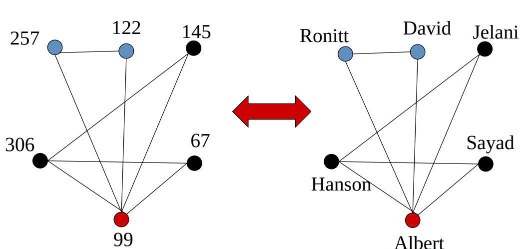
deg(a) = 2 deg(b) = 4

Connections not layout



Same graph, different layouts

Isomorphic Graphs



"same" graph, different labels

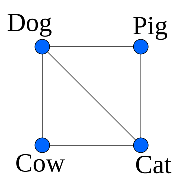
Graph Isomorphism

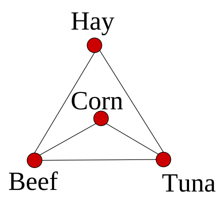
Graphs G_1 and G_2 are isomorphic if there exists a *bijection* $f: V_1 \to V_2$ such that for all u,v in V_1

u—v is in E_1 **iff** $f(u)$—$f(v)$ is in E_2

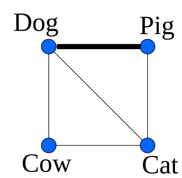
There is a one-to-one correspondence between the nodes of G_1 and G_2 that preserves all edge connections.

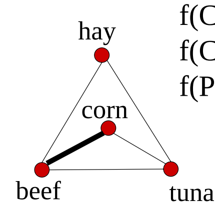
Are these Isomorphic?





Find a Mapping

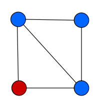


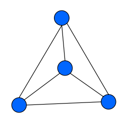


Function
f(Dog) = beef
f(Cat) = tuna
f(Cow) = hay
f(Pig) = corn

Finding the Mapping

- Not easy, can try all possible mappings
  - Roughly n! possibilities
- Can test for Invariants
  - Same number of nodes, edges
  - Same degree distributions
  - Preserves cycles, longest path, etc





Class Problems 1 & 2