

6	9	13	7
12		10	5
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Mathematics for Computer Science

MIT 6.042J/18.062J

# Truth & Proof

*Math vs. Reality*  
*Propositional Logic*

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Team Problem

# Surprise Problem 1

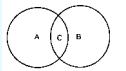
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## Math



Sets

Numbers  $4, \sqrt{7}, \pi, i + 1$

$t, f$

Booleans

Strings "albert meyer"

$$f(x) = x^2 + 2$$

Functions

Relations  $a \leq b$



Data structures

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## Not Math



Family

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## Not Math



Cats

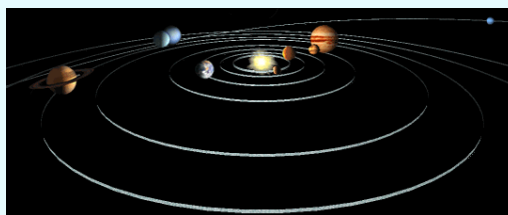
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## Not Math



Solar System

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## Not Math: *Cogito ergo sum*



**René Descartes'**  
MEDITATIONS

on *First Philosophy in which the Existence of God and the Distinction Between Mind and Body are Demonstrated.*

(<http://www.brainiacnet.com/~glynhughes/eqsahed/descartes.htm>)

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## Evidence vs. Proof

Let  $p(n) ::= n^2 + n + 41$ .

*Claim:*

$\forall n \in \mathbb{N}$   $p(n)$  is a prime number

For all  $n$  that are *natural numbers*

$0, 1, 2, \dots$

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## Only Prime Numbers?

Evidence:

$p(0) = 41$	prime	
$p(1) = 43$	prime	
$p(2) = 47$	prime	
$p(3) = 53$	prime	
$\vdots$		
$p(20) = 461$	prime	looking good!
$\vdots$		
$p(39) = 1601$	prime	enough already!

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lec 1E9

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## Only Prime Numbers?

$\forall n \in \mathbb{N}$   $p(n) ::= n^2 + n + 41$   
is a prime number

This is not a coincidence.

The hypothesis must be true. *But no!*

$p(40) = 1681$  is *not prime*.

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## Only Prime Numbers?

*Quickie:*

Prove that **1681** is not prime.

*Proof:*  $1681 = p(40)$   
 $= 40^2 + 40 + 41$   
 $= 40^2 + 2 \cdot 40 + 1$   
 $= (40 + 1)^2$

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## Evidence vs. Proof

**EULER'S CONJECTURE** (1769)

$$a^4 + b^4 + c^4 = d^4$$

has *no solution* for  $a, b, c, d$  positive integers

$$\forall a \in \mathbb{Z}^+ \forall b \in \mathbb{Z}^+ \forall c \in \mathbb{Z}^+ \forall d \in \mathbb{Z}^+$$

$$a^4 + b^4 + c^4 \neq d^4$$

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## Euler's Conjecture

**Counterexample:** 218 years later by Noam Elkies at Liberal Arts school up Mass Ave:

$$95800^4 + 217519^4 + 414560^4 = 422481^4$$

*Proof by computer:*

```
(= (+ (expt 95800 4)
      (expt 217519 4)
      (expt 414560 4))
   (expt 422481 4))
;Value: #t
```

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## Propositional (Boolean) Logic

*Proposition* is either **True** or **False**

Examples:  $2 + 2 = 4$       **True**  
 $1 \times 1 = 4$               **False**

Non-examples: Wake up!  
 Where am I?

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## Operators

$\wedge$  ::= AND  
 $\vee$  ::= OR  
 $\neg$  ::= NOT  
 $\rightarrow$  ::= IMPLIES  
 $\leftrightarrow$  ::= IFF (if and only if)

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## English to Math

“If Greeks are Human, and Humans are Mortal, then Greeks are Mortal.”

$$((G \rightarrow H) \wedge (H \rightarrow M)) \rightarrow (G \rightarrow M)$$

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## English to Math

Greeks carry Swords or Javelins

$$(G \rightarrow S) \vee (G \rightarrow J)$$

disjunction

*True even if a Greek carries both*

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## English to Math

Greeks carry Bronze or Flint swords

$$(G \rightarrow B) \oplus (G \rightarrow F)$$

exclusive-or

$P \oplus Q$  means “ $P$  or  $Q$  but **not both**”

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### Math vs. English

Parent: If you don't clean your room, you can't watch a DVD."

$\underbrace{\hspace{10em}}_C$

$\underbrace{\hspace{10em}}_D$

$\neg C \longrightarrow \neg D$  and

$C \longrightarrow D$

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### Math vs. English

Parent: If you don't clean your room, you can't watch a DVD."

$\underbrace{\hspace{10em}}_C$

$\underbrace{\hspace{10em}}_D$

*that is*

$C \longleftrightarrow D$

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### Math vs. English

Mathematician: "If a function is not continuous, then it is not differentiable."

$\underbrace{\hspace{10em}}_C$

$\underbrace{\hspace{10em}}_D$

$\neg C \longrightarrow \neg D$

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### Math vs. English

Mathematician: "If a function is not continuous, then it is not differentiable."

$\underbrace{\hspace{10em}}_C$

$\underbrace{\hspace{10em}}_D$

*But*  $C \longrightarrow D$

*is not implied*

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### Deductions

From:  $P$  implies  $Q$ ,  $Q$  implies  $R$

Conclude:  $P$  implies  $R$

*Antecedents*

$\underbrace{(P \rightarrow Q), (Q \rightarrow R)}_{P \rightarrow R}$

*Conclusion*

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### Sound Rules

*Definition:* A rule is *sound* if the conclusion is true whenever **all** antecedents are true.

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### A sound deduction

$$\frac{P \rightarrow Q, P}{Q}$$

Modus ponens

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### An Unsound Deduction

$$\frac{\bar{P} \rightarrow \bar{Q}}{P \rightarrow Q}$$

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### An Unsound Deduction

not Smart  $\rightarrow$  not MIT-student    **Yes!**

Smart  $\rightarrow$  MIT-student    **No!**

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### Team Problem

# Problems 2 & 3

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