Mathematics for Computer Science

MIT 6.042J/18.062J

Truth & Proof

Math vs. Reality

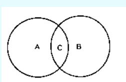
Propositional Logic

Team Problem

Surprise

Problem 1

Math



Sets

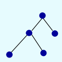
Numbers $4, \sqrt{7}, \pi, i+1$

t, f Booleans

Strings "albert meyer"

$f(x) = x^2 + 2$ Functions

Relations $a \leq b$



Data structures

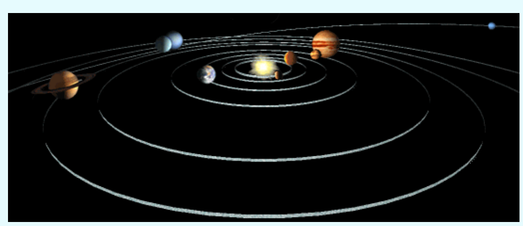
Not Math

Family

Not Math

Cats

Not Math



Solar System

Not Math: *Cogito ergo sum*

René Descartes'
MEDITATIONS
on First Philosophy in which the Existence of God and the Distinction Between Mind and Body are Demonstrated.

(http://www.btinternet.com/~glynhughes/squashed/descartes.htm)

Evidence vs. Proof

Let $p(n) ::= n^2 + n + 41$.

Claim:

$$\underbrace{\forall\, n}_{\text{For all } n} \underbrace{\in \mathbb{N}}_{\text{that are}}\ p(n) \text{ is a prime number}$$

For all n that are $\underbrace{\textit{natural numbers}}_{0,1,2,\ldots}$

Only Prime Numbers?

Evidence:

$p(0) = 41$	prime	
$p(1) = 43$	prime	
$p(2) = 47$	prime	
$p(3) = 53$	prime	
$\vdots$		
$p(20) = 461$	prime	**looking good!**
$\vdots$		
$p(39) = 1601$	prime	**enough already!**

Only Prime Numbers?

$$\forall\, n \in \mathbb{N}\ \ p(n) ::= n^2 + n + 41$$
is a prime number

This is not a coincidence.
The hypothesis must be true. *But no!*

$p(40) = 1681$ is *not prime*.

Only Prime Numbers?

Quickie:
Prove that 1681 is not prime.

Proof: 1681 = p(40)
$= 40^2 + 40 + 41$
$= 40^2 + 2\cdot 40 + 1$
$= (40 + 1)^2$

Evidence vs. Proof

EULER'S CONJECTURE (1769)

$$a^4 + b^4 + c^4 = d^4$$

has *no solution* for a,b,c,d positive integers

$$\forall a \in \mathbb{Z}^+ \forall b \in \mathbb{Z}^+ \forall c \in \mathbb{Z}^+ \forall d \in \mathbb{Z}^+$$
$$a^4 + b^4 + c^4 \neq d^4$$

Euler's Conjecture

Counterexample: 218 years later by Noam Elkies at Liberal Arts school up Mass Ave:

$$95800^4 + 217519^4 + 414560^4 = 422481^4$$

Proof by computer:

```
(= (+ (expt 95800  4)
      (expt 217519 4)
      (expt 414560 4))
   (expt 422481 4))
;Value: #t
```


Propositional (Boolean) Logic

Proposition is either **True** or **False**

Examples: $2+2=4$ True

$1\times 1=4$ False

Non-examples: Wake up!

Where am I?

Operators

$\wedge$ ::= AND

$\vee$ ::= OR

$\neg$ ::= NOT

$\rightarrow$ ::= IMPLIES

$\leftrightarrow$ ::= IFF (if and only if)

English to Math

"If Greeks are Human, and Humans are Mortal, then Greeks are Mortal."

$$((G \rightarrow H) \wedge (H \rightarrow M)) \rightarrow (G \rightarrow M)$$

English to Math

Greeks carry Swords or Javelins

$$(G \rightarrow S) \underbrace{\vee}_{\text{disjunction}} (G \rightarrow J)$$

True even if a Greek carries both

English to Math

Greeks carry Bronze or Flint swords

$$(G \rightarrow B) \underbrace{\oplus}_{\text{exclusive-or}} (G \rightarrow F)$$

$P \oplus Q$ means "P or Q but not both"

Math vs. English

Parent: If you don't $\overbrace{\text{clean your room}}^{C}$,
you can't $\underbrace{\text{watch a DVD}}_{D}$."

$$\neg C \longrightarrow \neg D \quad \textit{and}$$
$$C \longrightarrow D$$

Math vs. English

Parent: If you don't $\overbrace{\text{clean your room}}^{C}$,
you can't $\underbrace{\text{watch a DVD}}_{D}$."

that is

$$C \longleftrightarrow D$$

Math vs. English

Mathematician:
"If a function is not $\overbrace{\text{continuous}}^{C}$,
then it is not $\underbrace{\text{differentiable}}_{D}$."

$$\neg C \longrightarrow \neg D$$

Math vs. English

Mathematician:
"If a function is not $\overbrace{\text{continuous}}^{C}$,
then it is not $\underbrace{\text{differentiable}}_{D}$."

But $C \longrightarrow D$
is not implied

Deductions

From: P implies Q, Q implies R
Conclude: P implies R

$$\frac{\overbrace{(P \to Q),\ (Q \to R)}^{\textit{Antecedents}}}{\underbrace{P \to R}_{\textit{Conclusion}}}$$

Sound Rules

Definition: A rule is *sound* if the conclusion is true whenever all antecedents are true.

A sound deduction

$$\frac{P \rightarrow Q, \quad P}{Q}$$

Modus ponens

An Unsound Deduction

$$\frac{\overline{P} \rightarrow \overline{Q}}{P \rightarrow Q}$$

An Unsound Deduction

not Smart $\rightarrow$ not MIT-student	Yes!
Smart $\rightarrow$ MIT-student	No!

Team Problem

Problems 2 & 3

