In-Class Problems Week 5, Fri.

Problem 1. Figures 1–4 show different pictures of planar graphs.

(a) For each picture, describe its faces by listing the cycles that form their boundaries.

(b) Which of the pictured graphs are isomorphic? Which pictures represent the same (abstract) *planar drawing*?

Problem 2. Prove that in a drawing of a connected planar graph

(a) every edge is traversed exactly twice by the face boundaries.

(b) if the graph has 3 or more vertices, then every face has length at least 3 (more precisely, every face traverses edges at least 3 times).

Problem 3. Suppose we consider planar graphs that may *not* be connected. So in addition to the parameters v, e, f of Euler's formula, we also have the number, c, of connected components in the graph. In this case a face may have a boundary consisting of several unconnected cycles, as in Figure 5.

(a) Use the additional parameter, c, to state a generalized version of Euler's Formula that applies to possibly unconnected planar graphs.

(b) Drawings of unconnected graphs use two additional rules: rule (3): add a new vertex of degree 0, and rule (4) add an edge between vertices on two different connected components. Using these rules, explain how to modify the inductive proof of Euler's formula to prove the generalized version.

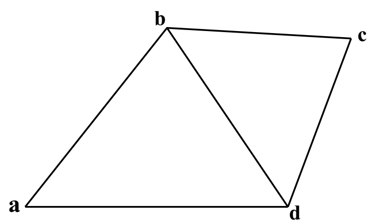


Figure 1

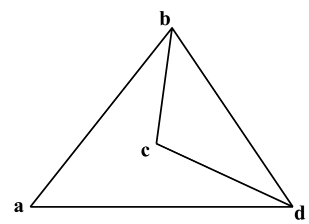


Figure 2

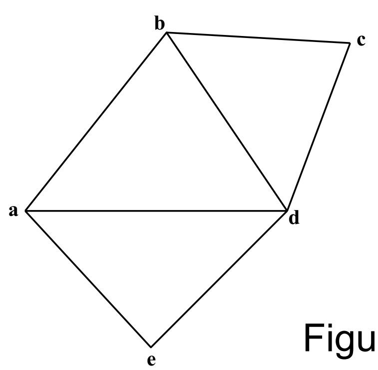


Figure 3

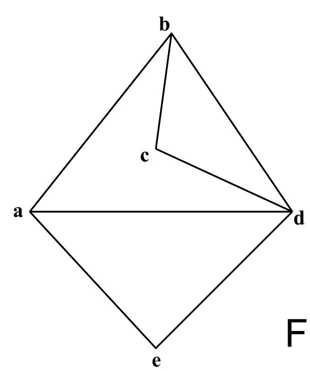
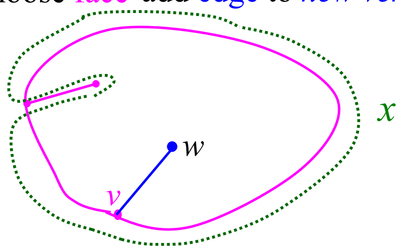


Figure 4

Face Creation Rules

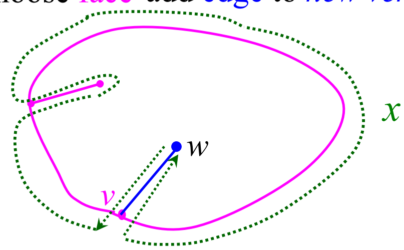
1) choose face add edge to *new vertex*



old face ~~*vxv*~~

Face Creation Rules

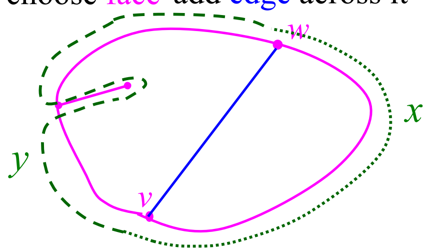
1) choose face add edge to *new vertex*



new face is *wvxvw*

Face Creation Rules

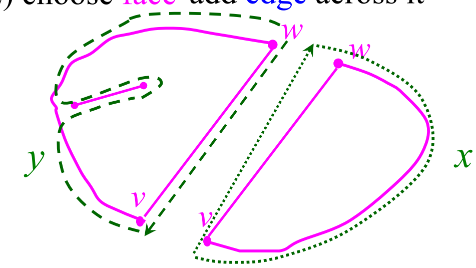
2) choose face add edge across it



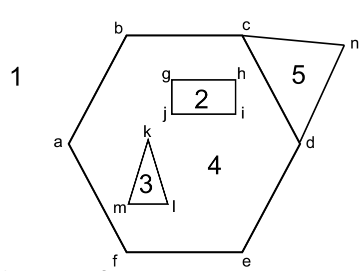
old face: ~~*wxvyw*~~

Face Creation Rules

2) choose face add edge across it



splits into 2 faces: *wxvw*, *vywv*



outer face 1: abcndefa
face 2: ghijg
face 3: klmk
face 4: {abcdefa, ghijg, klmk}
face 5: cdnc

Figure 5