In-Class Problems Week 1, Wed.

Problem 1. Identify exactly where the bugs are in each of the following bogus proofs.[1]

(a) $1/8 > 1/4$.

Bogus proof.

$$\begin{aligned} 3 &> 2 \\ 3\log_{10}(1/2) &> 2\log_{10}(1/2) \\ \log_{10}(1/2)^3 &> \log_{10}(1/2)^2 \\ (1/2)^3 &> (1/2)^2, \end{aligned}$$

and the claim now follows by the rules for multiplying fractions. □

(b) $1¢ = \$0.01 = (\$0.1)^2 = (10¢)^2 = 100¢ = \$1$.

[1] From Stueben, Michael and Diane Sandford. *Twenty Years Before the Blackboard*, Math. Assoc America, ©1998.

Problem 2.

Proposition (Arithmetic-Geometric Mean Inequality)**.** *For all nonnegative real numbers a and b*

$$\frac{a+b}{2} \geq \sqrt{ab}.$$

What is wrong with the following proof of this proposition?

Bogus proof.

$$\begin{aligned}
\frac{a+b}{2} &\overset{?}{\geq} \sqrt{ab} \\
a+b &\overset{?}{\geq} 2\sqrt{ab} \\
a^2+2ab+b^2 &\overset{?}{\geq} 4ab \\
a^2-2ab+b^2 &\overset{?}{\geq} 0 \\
(a-b)^2 &\geq 0
\end{aligned}$$

The last statement is true because $a-b$ is a real number, and the square of a real number is never negative. This proves the claim. □