# 15.081 Fall 2009
# Recitation
# for Lectures {9,10,11}
# Duality theory

Duality is the most important characteristic of LP. LP is useful and easy only because of the existence of strong duality. As one can observe, whenever we find a system where we find strong duality we can apply many of the algorithms that we develop for LPs for those problems. In particular, I recommend the following book for people interested in Duality theory. Duality in Optimization

## 1 Concepts

### Primal and Dual

1. Lagrangean Duality : The duality theory developed arises from consideing the lagrangean relaxation of the LP.

2. Weak Duality : Given any dual feasible $p$ and primal feasible $x$ suc that $p'b \leq c'x$ .

3. Strong Duality and Complementary Slackness - The properties of optimal solutions of primal and dual.

### Geometry

1. Farkas Lemma, which turns out to be equivalent to Strong Duality.

2. Cones and Extreme Rays - Extreme rays of a polyhedron correspond to the extreme rays of the recession cone assocciated with the polyhedron.

3. Resolution theorem : Another representation of a polyhedron.

**Duality and Degeneracy**

An important relation that can be useful is the following :

- Uniqueness and Nondegeneracy of primal $\iff$ Uniqueness and Nondegeneracy of the dual.

This would thus imply that :

- Non-uniqueness or degenracy of primal $\iff$ Non-uniqueness or degenracy of the dual.

**Proof Techniques**

The proof techniques that duality or Farkas' Lemma provides us with are very useful.In particuar, duality allows us to convert an "existential statement" into a "for all statement". Whenever we have an "existential statement" in the hypothesis, duality would give us "for all statements" as conclusions and vice-versa. Most of the exercises are (slighlty non-trivial) applications of this proof technique.

In particular, to use duality we have identify

- Appropriate Primal Polyhedron - This usually is very apparent from the statement of the problem that we are considering.
- Appropriate cost function - this requires "clever" choice and usually choices of $c = 0, \pm e, \pm e_i$ work.

**Examples :**

1. Strict complementary slackness (4.20) - The polyhderon and the cost vector suggested in the hint.
2. Clarke's theorem - Polyhedron is obvious. cost vector - Use $-e$.
3. 4.26 : USe $e$

6.251J / 15.081J Introduction to Mathematical Programming
Fall 2009