6.254 : Game Theory with Engineering Applications
Lecture 1: Introduction

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February 2, 2010

Introduction

- Optimization Theory: Optimize a single objective over a decision variable $x \in \mathbb{R}^n$.

$$\begin{array}{rc} \text{minimize} & \sum_i u_i(x) \\ \text{subject to} & x \in X \subset \mathbb{R}^n. \end{array}$$

- Game Theory: Study of multi-person decision problems
  - Used in economics, political science, biology to understand
    - competition and cooperation among agents.
    - role of threats/punishments in long term relations.
  - Models of adversarial behavior (strictly competitive strategic interactions, modeled as zero sum games).
    - Pursuit-evasion games.

Introduction

- Game Theory (Continued):
  - Recent interest in networked-systems (communication and transportation networks, electricity markets).
    - Large-scale networks emerged from interconnections of smaller networks and their operation relies on various degrees of competition and cooperation.
    - Online advertising on the Internet: Sponsored search auctions.
    - Distributed control of competing heterogeneous users.
    - Information evolution and belief propagation in social networks.
- Model: n agents , each chooses some $x_i \in \mathbb{R}$, and has a utility function $u_i(x)$, $x \in \mathbb{R}^n$, or equivalently

$$u_i(x_i, x_{-i}), \qquad x_{-i} = (x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n).$$

  - What are the possible outcomes?
  - Steady-state, stable operating point, characteristics?
  - How do you get there (learning dynamics, computation of equilibrium)?

Introduction

- Mechanism Design (MD): "Inverse Game Theory": Design of a game (or incentives) to achieve an objective (eg. system-wide goal or designer's selfish objective)
  - Optimization theory extended for systems in which there are independent agents not under direct control, and must be "coerced" through the use of incentives.
  - Focal example: Internet
  - Users' interest to skimp on congestion control
  - ISP's interest to lie about routing information.
- In Economics, MD is all about designing the right incentives.
- In CS/Engineering, focus is more on the design of efficient decentralized protocols that take into account incentives.

Course Information

- Introduction to fundamentals of game theory and mechanism design.
- Emphasis on the foundations of the theory, mathematical tools; modeling issues and equilibrium notions in different environments.
- Motivations drawn from various applications:
    - Engineered and networked systems: including distributed control of wireline and wireless communication networks, incentive-compatible and dynamic resource allocation, multi-agent systems, pricing and investment decisions in the Internet.
    - Social models: including learning and dynamics over social and economic networks.
- Intended Audience: The course is geared towards Engineering-OR-CS students who need to use game-theoretical tools in their research. The course is also aimed at covering recent advances and open research areas in game theory.

Course Information

- Prerequisites: A course in probability (6.041 equivalent) and mathematical maturity. A course in analysis (18.100 equivalent), and a course in optimization (6.251-6.255 equivalent) would be helpful but not required.
- Grading:
    - 30 %: midterm
    - 20 %: homeworks
    - 50 %: project
- Project: Individual or groups of 2. Possible project types include but are not limited to:
    - Read and report on 2-4 papers on a theoretical/application area related to game theory.
    - An experimental study via implementation and simulation of a game/mechanism.
    - Theoretical analysis of a game-theoretic model, which we have not covered in class and which has not been fully explored in the literature.
- As a starting point, check the reading list on the website.

Text and References

- Main Text: Game Theory, by D. Fudenberg and J. Tirole, MIT Press, 1991.
- Other Useful References: The class notes available on the web.
  - Algorithmic Game Theory, edited by N. Nisan, T. Roughgarden, E. Tardos, and V. V. Vazirani, Cambridge University Press, 2007.
  - Auction Theory, by V. Krishna, Academic Press, 2002.
  - Microeconomic Theory, by A. Mascolell, M. D. Whinston, and J. R. Green, Oxford University Press, 1995.
  - A Course in Game Theory, by M.J. Osborne, A. Rubinstein, MIT Press, 1994.
  - Game Theory, R. B. Myerson, Harvard University Press, 1991.
  - The Theory of Learning in Games, by D. Fudenberg and D. Levine, MIT Press, 1999.
  - Strategic Learning and its Limits, by H.P. Young, Oxford U Press, 2004.
  - Individual Strategy and Social Structure: An Evolutionary Theory of Institutions, by H. P. Young, Princeton University Press, 1998.
  - Dynamic Noncooperative Game Theory, by T. Basar and G. J. Olsder, 1999.

Strategic Form Games

- Model for static games.
- Both matrix games and continuous games.
- Classical examples as well as examples from networking: "Selfish routing", resource allocation by market mechanisms, inter-domain routing across autonomous systems.
- Solution concepts:
    - Dominant and dominated strategies
    - Elimination of strictly dominated strategies (iterated strict dominance).
    - Elimination of never-best-responses (rationalizability).
    - Nash equilibrium; pure and mixed strategies; mixed Nash equilibrium.
    - Correlated equilibrium (Aumann).

Analysis of Static (Finite and Continuous) Games

- Existence of a pure and mixed equilibrium
  - Nash Equilibrium: fixed point of best-response correspondences.
  - Nash's theorem (for finite games, use fixed-point theorems to show existence of a mixed strategy Nash equilibrium)
  - For continuous games: under convexity assumptions, can show existence of a pure strategy Nash equilibrium.
  - For general continuous games, can show existence of a mixed strategy equilibrium.
  - For discontinuous games (relevant in models of competition), existence of a mixed equilibrium established under some assumptions.
- Uniqueness of an equilibrium using "strict diagonal concavity" assumptions

Games with Special Structure

- Supermodular games: Instead of convexity, we have some order structure on the strategy sets of the players and conditions which guarantee "increase in strategies of the opponents of a player raises the desirability of playing a high strategy for this player."
  - Nice properties: Existence of a pure strategy equilibrium, convergence of simple greedy dynamics (strategy updates) to a pure strategy Nash equilibrium, lattice structure of the equilibrium set.
  - Recent applications in wireless power control.
- Potential games: Games that admit a "potential function" (as in physical systems) such that maximization with respect to subcomponents coincide with the maximization problem of each player.
  - Similar nice properties.
  - Relation to congestion games: "Payoff of a player playing a strategy depends on the total number of players playing the same strategy"
  - Recent applications in network design games.

Learning, Evolution, and Computation(Finite Games)

- Learning:
  - Best-response dynamics, fictitious play (i.e., play best-response to empirical frequencies), dynamic fictitious play; convergence to Nash equilibrium.
  - Regret-matching algorithms; convergence to correlated equilibrium.
- Evolution:
  - Evolutionarily stable strategies.
  - Replicator dynamics and convergence.
- Computation of Equilibrium:
  - Zero-sum games.
  - Nonzero-sum games.
  - Algorithms that exploit polyhedral structure, Lemke-Howson algorithm; algorithms for finding fixed-points, Scarf's algorithm; exhaustive "smart" search etc.

Extensive Form Games and Repeated Games

- Multi-stage games with perfect information:
  - Backward induction and subgame perfect equilibrium.
  - Applications in bargaining games. Nash bargaining solution.
- Repeated games:
  - Infinitely and finitely repeated games, sustaining desirable/cooperative outcomes (e.g. Prisoner's Dilemma)
  - Trigger strategies, folk theorems
  - Imperfect monitoring and perfect public equilibrium.
- Stochastic games
  - Markov strategies and Markov perfect equilibrium.

Games with Incomplete Information and Introduction to Mechanisms

- Static games with incomplete information.
  - Bayesian Nash Equilibrium
  - Each player has private information (called his "type"). Players know the conditional distribution of types of other players.
- Extensive form games with incomplete information
  - Perfect Bayesian Equilibrium
- Applications in auctions:
  - Different auction formats (first-price, second-price auctions)
  - Revenue and efficiency properties of different auction formats
  - Can we design the "optimal" auction for a given objective?

Mechanism Design

- Design of game forms to implement certain desirable outcomes.
  - e.g. to incentivize independent agents to reveal their types truthfully.
- Mechanism as a mapping that maps "signals" from independent agents into allocations and payments (or transfers)
- Revelation principle, incentive compatibility
- Optimal Mechanisms (Myerson): Design a mechanism to maximize profits.
- Efficient Mechanisms (Vickrey-Clarke-Groves Mechanisms): Design a mechanism to maximize a "social" or system-wide objective.
  - Mechanisms in networks; distributed and online mechanisms.
  - Mechanisms that operate with limited information

Network Effects and Games over Networks

- Positive and negative externalities.
- Utility-based resource allocation: congestion control.
- Selfish routing. Wardrop and Nash equilibrium.
- Partially optimal routing.
- Network pricing: Combined pricing and traffic engineering.
  - Competition among service providers and implications on network performance.
- Strategic network formation.
- Price of anarchy: Game-theory analogue of "approximation bounds"
  - Ratio of performance of "selfish" to performance of "social".

6.254 Game Theory with Engineering Applications
Spring 2010