Problem 1 *(Three player game)*
Consider the following game with three players. Player 1 chooses row; player 2 chooses column; and player 3 chooses matrix.

P1 \ P2	L	R
U	1,1,1	0,0,0
D	0,0,0	0,0,0

P1 \ P2	L	R
U	0,0,0	0,0,0
D	0,0,0	1,1,1

(a) What is the minmax level?

(b) What is the set of feasible payoffs?

(c) For any $\delta \in (0, 1)$, show that there is no SPE of G^∞ with average payoff less than $\frac{1}{4}$.

Problem 2 *(Cooperation over social network)*
Consider a social network, in which agents are matched pairwise at each date according to a matrix of probabilities P (where all entries of P are strictly positive). Once matched, each pair plays the prisoner's dilemma game with the following payoffs.

prisoner 1 / prisoner 2	Cooperate	Defect
Cooperate	1, 1	-1, 2
Defect	2, -1	0, 0

Table 1: Prisoner's Dilemma

(a) Consider trigger strategies in which each player chooses its actions against a specific opponent only according to the history of their own past interactions. Show that for any discount factor $\delta < 1$, there exist matching probabilities such that players will not be able to support cooperation with more than one other player.

(b) Suppose now that strategies are function of the specific opponents entire history (against each of his past opponents). In this case, show that cooperation against all players can be supported for $\delta \geq 1/2$.

Problem 3 *(Imperfect information Bertrand)*
Consider the Bertrand game in which there is a one unit demand with reservation price R greater than or equal to 1 and each firm has a marginal cost c_i drawn uniformly between [0, 1]. Find the Bayesian Nash equilibria of this game.

Problem 4 *(War of attrition)*
In a war of attrition, two players compete for an object by expending resources over time (i.e., time is valuable). The value of the object to player i is $v_i > 0$. Consider an incomplete-information version of the war of attrition of two players, in which the values are privately known. Player i chooses the time to concede $t_i \in [0, +\infty)$ according to his/her strategy $s_i(v_i) = t_i$, where v_i is player i's value of the object, which takes value in $[0, +\infty)$ with cumulative distribution function F and probability density function f. The valuations are independent between players. Both players choose simultaneously. The payoffs are

$$u_i(t_1, t_2) = \begin{cases} -t_i & \text{if} \quad t_i < t_j \\ v_i/2 - t_i & \text{if} \quad t_i = t_j \\ v_i - t_j & \text{if} \quad t_i > t_j \end{cases}$$

(a) Characterize a symmetric Bayesian Nash equilibrium.

(b) Let $F(v) = 1 - \exp(-v)$ for $v \geq 0$. Derive the symmetric Bayesian Nash equilibrium explicitly.

(c) Relate the war of attrition with incomplete information to the auction forms that we have seen. Assume that there exists a symmetric equilibrium $s(v)$, increasing in v, such that the expected payment of a bidder with value 0 is 0. Use the revenue equivalence theorem to derive a symmetric Bayesian Nash equilibrium.

Problem 5 *(Reserve prices)*
In many instances, sellers reserve the right to not sell the object if the price determined in the auction is lower than some threshold amount, say $r > 0$. Such a price is called *reserve price*. In this problem, we would like to examine the effects of such a reserve price on the expected revenue accruing to the seller.

(a) (*Reserve Prices in a Second-Price Auction*) What is the expected payment of a bidder with value x in a second-price auction with reservation price r?

(b) (*Reserve Prices in a First-Price Auction*) What is the expected payment of a bidder with value x in a first-price auction with reservation price r?

(c) (*Revenue Effects of Reserve Prices*) What is the optimal, or revenue maximizing, reserve price from the perspective of the seller?

Problem 6 *(SPE in Folk Theorem)* **[Bonus]**
Verify that the strategy constructed in the proof for the Subgame Perfect Folk Theorem by Fudenberg and Maskin is an SPE.

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