Last Time:

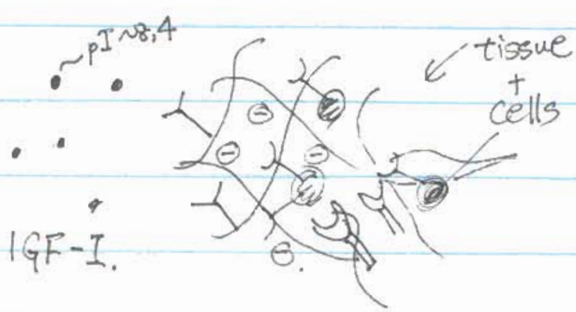
1. Flux
2. Continuity
3. Boundary Cond.
4. K (partition coeff)
5. Example: Steady State Diffusion } tumor + chemoattractant

$$\cancel{\left(\frac{\partial c}{\partial t}\right)}^{0} = D\nabla^2 C + \cancel{R}^{0} = 0$$

Today: 1) Transient Diffusion ($R=0$)

2) Separation of variables

3) Ex: Diff of IGF-I thru [tissue/membrane].



- want to "deliver" IGF-I for stimulating growth, repair
- In general; IGF-I can bind to
  - a) cell receptors Ψ
  - b) non-specific bind to ECM Y
  - c) specific binding proteins sequestered in ECM "IGF-BPs"

Objectives: (1) Char diff. transport of IGF-I into & thru tissue

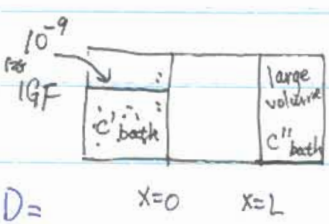
2) measure "D"

3) Measure binding parameter

4) Add convection & elec. migration

Exp + Model:

- Transport chamber
- use ^{125}I-IGF
- Assume: $R=0$; $K=1$; $D=$ const (indp of concentration)



(1) $N_x = -D\frac{\partial c}{\partial x}$

(2) $\frac{\partial c}{\partial t} = D\frac{\partial^2 c(x,t)}{\partial x^2}$

I.C.: $c(x,t=0)=0$ $\quad$ ($k \sim 1.4$ IGF-I)

B.C.: (a) $c(x=0,t) = kc' \equiv c' \quad (k \equiv 1)$

$c(x=L,t) = kc'' = c'' = 0$? or $c(t)$

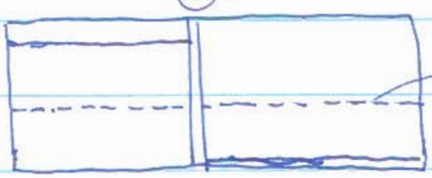
steric keep partition ↓

electrostatic keep part. ↑

Find $c(x,t)$ for $\begin{cases} 0 < x < L \\ 0 < t < \infty \end{cases}$

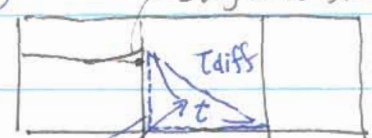
stirrer keeps concentration uniform.

③.



$\tau_{chamber} \gg \tau_{diff} \sim \dfrac{L^2}{(\pi^2)D}$

②. stagnant film.



large t compare to diffusion

→ high flux.

$t = 0^+$

"large t" ⇒

st. state: $c(x) = c'\left(1 - \frac{x}{L}\right)$

Method: Separation of Variables

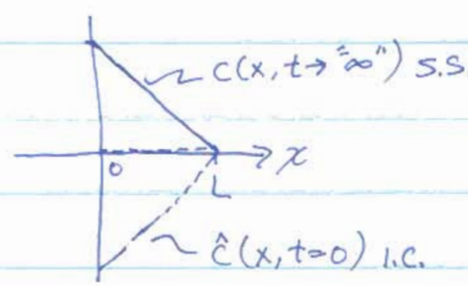
Let: $\underbrace{c(x,t)}_{\text{want}} = \underbrace{\hat{c}(x,t)}_{\text{decays in time.}} + \underbrace{c'\left(1-\frac{x}{L}\right)}_{\text{st. state}}$

Find: $\hat{c}(x,t)$: sol'n to $\dfrac{\partial \hat{c}(x,t)}{\partial t} = D \dfrac{\partial^2 c(x,t)}{\partial x^2}$

B.C.s $\left.\begin{aligned} \hat{c}(x=0,t) &= 0 \\ \hat{c}(x=L,t) &= 0 \end{aligned}\right\}$ homogeneous B.C.'s.

I.C. on $\hat{c}(x,t=0) = \underbrace{c(x,t)}_{=0}^{=0} - c'\left(1-\frac{x}{L}\right)$

$\hat{c}(x,t=0) = -c'\left(1-\frac{x}{L}\right)$ @ $t=0$



Separation of variables:

(A) Setup a product soln

$$\hat{C}(x,t) = X(x)\,T(t)$$

(B) substitute in PDE:

$$X(x)\frac{\partial T(t)}{\partial t} = D\,T(t)\frac{\partial^2 X}{\partial x^2}$$

(C)

$$\underbrace{\frac{1}{DT(t)}\left(\frac{\partial T}{\partial t}\right)}_{f(t\text{ alone})} = \underbrace{\frac{1}{X(x)}\frac{\partial^2 X(x)}{\partial x^2}}_{g(x\text{ alone})} = -k^2$$

(D) Solve + find values for k^2

$$\frac{\partial T(t)}{\partial t} = -k^2 D\,T(t) \Rightarrow T(t) = Ae^{-k^2Dt}$$

$$\frac{\partial^2 X}{\partial x^2} + k^2 X(x) = 0$$

$$X(x) = C_1\cos kx + C_2\sin kx$$

@ $x=0 \Rightarrow C_1 = 0$

$x = L \Rightarrow X(x) = C_2 \sin\frac{n\pi x}{L}$

$$\therefore \hat{C}(x,t) = X(x)\,T(t)$$

$$= \sum_{n=1}^{\infty} A_n' \sin\left(\frac{n\pi x}{L}\right) e^{-t\left(\frac{n^2\pi^2 D}{L^2}\right)}$$

find A_n