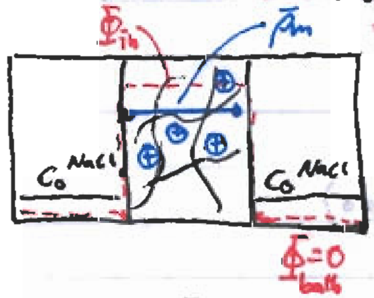
Today: Electromechanical Interaction Forces at Molecular, (Cell), Tissue Levels

Ⓘ Tissue: Donnan Osmotic Swelling
(macro-continuum scale & model)

Examples: Cornea, Tendon, Cartilage, ... (many)

Ⓘⓘ Molecular Repulsive Interactions (DLVO)
(nano-scale: Poisson-Boltzmann (PB))

Ⓘ DONNAN SWELLING (OSMOTIC)



- assume $\bar{\rho}_m = \text{const.}$
$\Rightarrow \bar{\Phi}^{in}$ is const.
$\Rightarrow E^{in} = -\nabla\bar{\Phi} = 0$!

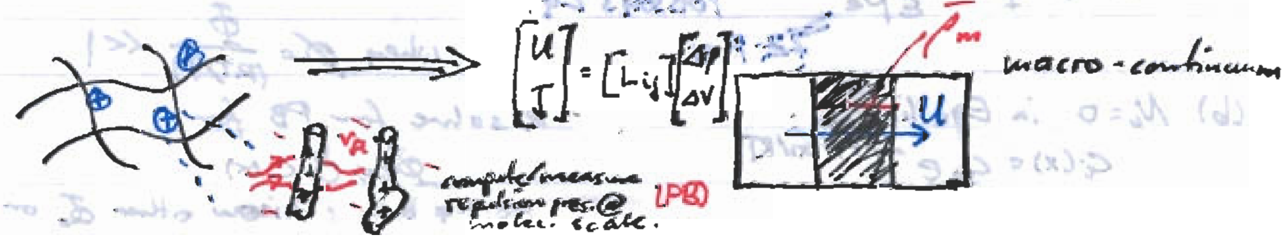
Given: $C_0, \bar{\rho}_m$

Find: $\bar{c}_+, \bar{c}_-$; $\Delta\pi^{os} = f(\bar{c}_+, \bar{\rho}_m)$

$\Delta I, \Delta pH$ → (C) → (E) → (M)

transient swelling
⇒ final equilibrium

- apply relations @ both macro- & molecular levels
→ have to be careful about assumptions at both sides.



① Donnan:
(a) $\rho_e \simeq 0$ (electroneut.)
(b) Boltzmann

$$\Rightarrow \quad \bar{c}_{\pm} = \mp\left(\frac{\rho_m}{2F}\right) + \sqrt{\left(\frac{\rho_m}{2F}\right)^2 + c_0^2}$$

Constitutive Law: van't Hoff

$$\pi^{os} = RT \sum_i c_i$$

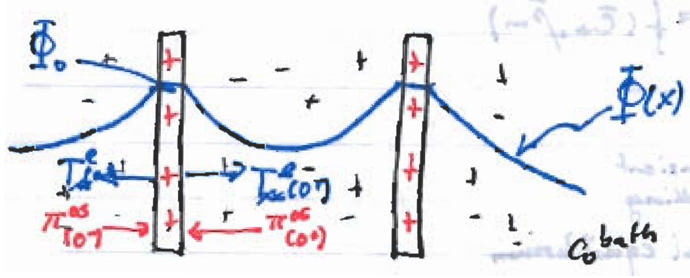
$$\Rightarrow \Delta\pi^{os} = RT(\bar{c}_+ + \bar{c}_- - 2c_0) \text{ "swelling"} \qquad (c)$$

$$\boxed{\Delta\pi^{os} = RT\left(2\sqrt{\left(\frac{\bar{\rho}_m}{2F}\right)^2 + c_0^2} - 2c_0\right)}$$ Cons

$\lim \bar{\rho}_m$: $\Delta\pi^{os} \to 0$

$$\left(\frac{\bar{\rho}_m}{F}\right) \gg c_0 \quad \Rightarrow \quad \Delta\pi = RT\left(\frac{\bar{\rho}_m}{F} - 2c_0\right)$$

② Nano-molecular Model: PB eqn. (Chapt. 4 §4.5-7)



Find: f^e causing repulsion.

know c_0 $1/\kappa$ inside $\sim 0.14_{nm}$

(a) Gauss ($\rho_e = \nabla \cdot \varepsilon \underline{E}$)
Faraday ($\underline{E} = -\nabla\Phi$)

$$\Rightarrow \nabla^2\Phi = -\frac{1}{\varepsilon}\rho_e \quad \text{Poisson's Eq.}$$

$\sum z_i F c_i$

(b) $N_i = 0$ in Equilibrium:

$$c_i(x) = c_{i0}\, e^{-z_i F \Phi(x)/RT}$$

$\Rightarrow$ PB

$$\nabla^2\phi = K^2 \sinh\phi$$

$$\nabla^2\phi = K^2\phi \quad \text{(linearized)}$$

when $\phi = \frac{\Phi}{(RT/F)} \ll 1$

$\Rightarrow$ solve for PB for $\phi(x)$ & $c(x)$

using B.C : know either Φ_0 or σ_d

(a) + (b) $\Rightarrow$ Solve P.B.

for $\Phi(x)$ & $c_i(x)$

$\Rightarrow$ solve for $E_x = -\frac{d\Phi(x)}{dx}$

Now (c) Stokes: $\cancel{\rho \frac{Dv}{Dt}}^{\to 0} = -\nabla p + \cancel{\mu \nabla^2 \underline{v}}^{\to 0} + \rho_e E$

In equil.

Balance: $0 = \underbrace{-\nabla p}_{\text{osmotic}} + \underbrace{\rho_e E}_{\text{elec. pressure}} \Rightarrow \nabla \cdot \underline{\underline{T}}^e$

between elec. & osmotic stresses at nano-scale

$$f_{net} = \oint_S \underline{\underline{T}} \cdot \overset{\text{normal}}{\underline{n}}\, ds$$