11/15/04

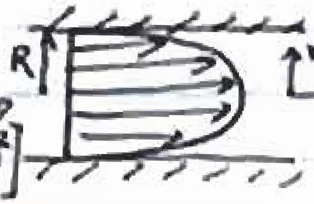
$\boxed{\text{Today}}$ Beginning of the home stretch :)
→ Integrative Case Studies

Ⓘ "Electrokinetic" Interactions - Electromechanics @ the μm - + nm - length scales

Ⓘ Applications to Ⓐ MEMS, NEMS; μm- fluidics
Ⓑ Membrane x-port
Ⓒ Mechanobiology

Ⓒ ← Ⓔ
Ⓜ

$\boxed{\text{Last Time}}$:



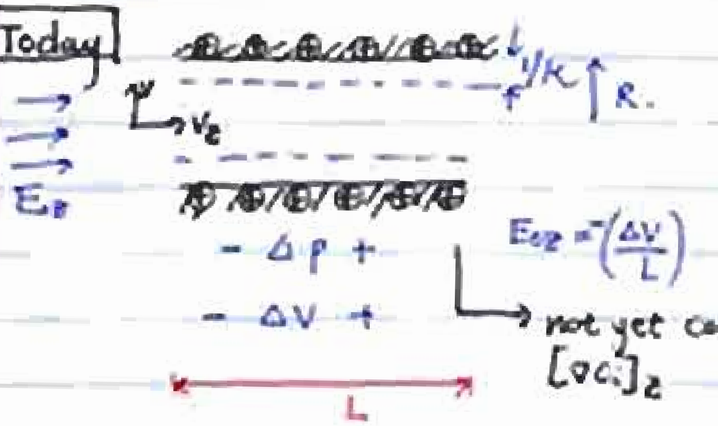
$\underline{v} = \hat{\imath}_z U\left[1-\left(\frac{r}{R}\right)^2\right]$

$\rho\left(\frac{\partial v}{\partial t} + \cancel{\underline{v}\cdot\nabla v}\right) = -\nabla p + \mu\nabla^2\underline{v} + \ldots$

A) Flow Regime. (i) "Fully-developed" flow $\underline{v} = \hat{\imath}_z v_z(r)$

B) Low Re: $Re = \frac{|\rho(Dv/Dt)|}{|\mu\nabla^2\underline{v}|} \sim \frac{\rho U L}{\mu} \ll 1$

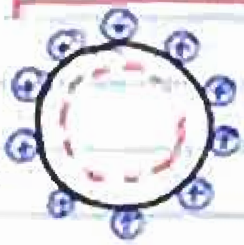
$\boxed{\text{Today}}$

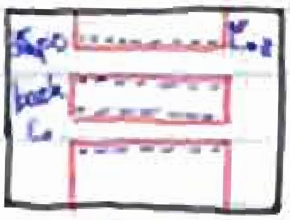


$E_{oz} = -\left(\frac{\Delta V}{L}\right)$

→ not yet considering $[\nabla c_i]_z$

Find: $\boxed{v_z(r) = (\text{"poiseuille"})\frac{\Delta p}{L} + \left(\text{"Electrical osmosis"}\right)\frac{\Delta V}{L}}$





Midterm Problem

Initial $\Phi(r)$ in Equilibrium for $t<0$ before E_{oz} is turned on.

concentration of ionic species varies spatially.

General: (1) $\underset{\text{ions}}{\underline{N_i}} = -D_i \nabla c_i + \frac{z_i}{|z_i|}\mu_i E_i^{(TOT)} + c_i \underline{v}$

(2) $\frac{\partial c_i}{\partial t} = -\nabla \cdot \underline{N}_i + R$ (ionization of pore charge)

(3) $\nabla \cdot \varepsilon \underline{E}^{(TOT)} = \rho_e = \sum_i z_i F c_i \neq 0$ Gauss Law, not electroneutrality

(4) $\underline{E}^{(TOT)} = -\nabla \Phi$ Faraday.

(5) $\nabla \cdot J = -\frac{\partial \rho_e}{\partial t} \approx 0$ $[\tau_{\text{relax}} \sim 1\text{ns}]$

(6) $\underline{J} = \sigma \underline{E}^{(TOT)} + \rho_e \underline{v} + \cancel{(\quad)\nabla c_i} \rightarrow 0$ today.

net mobile ions moving due to convection.

(7) $\rho_{mass}\cancel{\left[\left(\frac{\partial \underline{v}}{\partial t}\right) + \underline{v}\cdot\nabla \underline{v}\right]} \rightarrow 0 = -\nabla p + \mu \nabla^2 \underline{v} + \rho_e \underline{E}^{(TOT)} = 0$ Navier Stokes.

ions moving, passing their momentum to the fluid.

(8) $\nabla \cdot \underline{v} = 0$ (conservation of mass for incompressible fluid).

Assumptions / Approx.

(a) Newtonion, fully developed, Low Re flow.

$$Re = \frac{\rho U L}{\mu} = (10^6)(10^{-6} \to 10^{-3})(10^{-9} \to 10^{-4}) \ll 1$$

(b) Fully Developed: $\underline{v} = \hat{i}_z v_z(r)$

$\underline{J} = \hat{i}_z\left[\underbrace{\sigma(r) E_{oz} + \rho_e(r)}_{\text{both are function of concentration of species.}} v_z(r)\right]$ this form is self-consistent with assumption of fully-developed flow.

Initial Condition: $t<0$, $E_{oz}=0$, $\Delta p = 0$, $v_z = 0$.

Find $c_i(r,z)$ ⇓, $\Phi(r,z)$ $\Rightarrow$ (1) $\rightarrow c_i(r) = c_{io} e^{-z_i F \Phi(r)/RT}$ Boltzmann.

(3) + (4) $\Rightarrow$ P. B. $\Rightarrow \Phi(r) \Leftarrow \left\{\nabla^2 \Phi = -\frac{\rho_e}{\varepsilon}\right\}$

Radial Balance of Force:

$$\Big[\underbrace{\frac{1}{r}\frac{\partial}{\partial r} r(\varepsilon E_r)}_{\nabla\cdot\varepsilon E}\cdot \underline{\underline{E_r}} - \frac{\partial p}{\partial r}\Big] = 0 = \nabla\cdot\left(\underline{\underline{T}}^{e} + \underline{\underline{T}}^{p}\right)$$

What makes cells membrane move @ initial state?

- Electrostatic causes dipole repulsion.
- Different concentrations of each respective species causes osmotic pressure.
- Balance of electrostatic & osmotic pressure determine the Mechanical state, only happens when membrane is elastic.

Now: Turn on Applied E_{0z} &/or Δp

→ assume, still, that assump (b) is valid!

with flow: flow does not alter radial dependence of $C_i(r)$

" " " " " " of $\rho_e(r)$

" " " " " " " $\Phi(r)$

E(r

$$E_r \simeq \frac{0.1 \to 1 \text{ volt}}{10^{-9}\, m} \sim 10^8 - 10^9 \frac{V}{m} \;!!!$$

$$E_z \sim \frac{100V}{cm} \sim 10^3 \frac{V}{m} \sim 10^4 \frac{V}{m} \lll E_r \;!!!$$ (*)

Due to the unbalance of E_r, E_z, assumption of fully developed flow ok.

Now Find $V_z(r)$: (8) $\nabla\cdot \underline{v} = 0$ ✓

$$[(7)]_z \quad \frac{\partial p}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\underbrace{r\left[\frac{\partial V_z}{\partial r}\mu + \varepsilon E_r E_{0z}\right]}_{[\mu\nabla^2 \underline{v}]_z}$$

$$\downarrow$$

$$\left(\frac{\Delta p}{L}\right)\frac{r^2}{2} = r\left[\mu\frac{\partial V_z}{\partial r} + \varepsilon E_r\left(-\frac{\Delta V}{L}\right)\right] + C_1$$

BC's

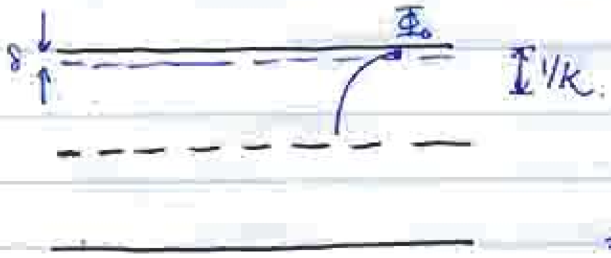
At r=0 (center)

$$\frac{\partial V_z}{\partial r} = 0 = E_r$$

$$\Rightarrow C_1 = 0$$

$$\left(\frac{\Delta P}{L}\frac{r^2}{4}\right) = \mu v_z(r) + \varepsilon\left(-\Phi(r)\right)\left(-\frac{\Delta V}{L}\right) + C_2$$

$$v_z(r) = \frac{r^2}{4\mu}\left(\frac{\Delta P}{L}\right) + \frac{\varepsilon}{\mu}\Phi(r)\left(-\frac{\Delta V}{L}\right) + C_2$$



@ $r = R - \delta$

a few H_2O molecules

$v_z \to 0$ (no slip)

$\Rightarrow$ "δ" potential!