Today: Begin Mechanical Subsystem

→ Selected Topics in Newton Fluid Mechs.

11/8/04

1. Conservation of Mass & Momentum
2. Define (Viscous) stress tensor $T_{ij} = C_{ijkl}\, \dot{e}_{ij}$
3. Viscous Dominated Flows:

   Define strain rate tensor $\dot{e}_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right)$

4. Relate T_{ij} to $\dot{e}_{ij}$ ⇒ Force density $\underline{F} = \nabla \cdot \underline{\underline{T}}$
5. Arrive @ Navier Stokes Eqn.

Characteristic Times & Lengths Scales

$L \sim \frac{1}{\kappa}$

$\tau_{diff} \simeq \frac{L^2}{(\pi^2) D_i}$ ≃ $\tau_{ch.\,relax} \sim \left(\frac{\varepsilon}{\sigma}\right)$

C ⇄ E

$\tau_{diff} \sim \frac{L^2}{(\ )D_{eff}}$

M

$\tau_{elec.\,diff.} \sim \frac{L^2}{(\ )D_i} \frac{1}{\left[1 + \left(\frac{E_o L}{(RT/F)}\right)^2\right]}$

$D_{eff} \sim \frac{D_i}{\left[1 + \frac{n K_d}{(K_d + C_o)^2}\right]}$

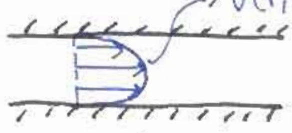
Add M: $\tau_{conv} = \frac{L}{v}$;

$\tau_{visc,\,diff} \sim \frac{L^2}{(\ )\frac{\mu}{\rho}}$; ....

ρ fluid mass density

Mech. Subsystem: "Low Reynold's number," "Viscous dominated", fully developed flow

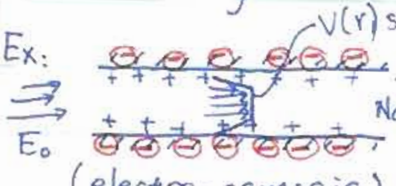
Ex:



"poiseuille flow"

+ $\Delta\phi$ −

Ex:

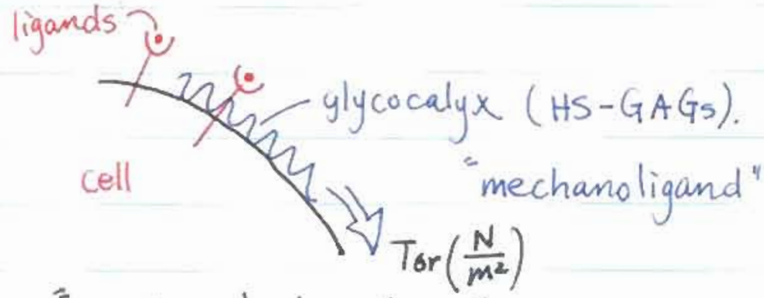


$\frac{1}{\kappa} \sim 1\,\mu m$

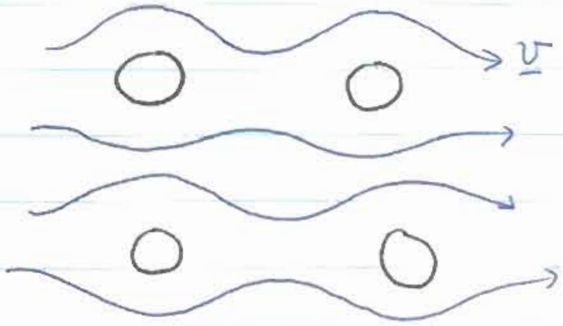
$R \sim 10\,\mu m$.

for $R \gg \left(\frac{1}{\kappa}\right)$ ⇒ "plug flow" @ $t = 0$

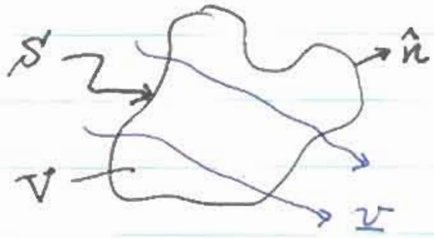
(electro-osmosis)



"mechanobiology" → "mechanotransduction"



1a). Conservation of mass



ρ = mass density $\left(\frac{Kg}{m^3}\right) \sim 10^3$ water

$\underline{v}$ = fluid density

$$\underbrace{\frac{d}{dt}\int_V \rho\, dV}_{\text{rate of acc. of mass in V.}} = \underbrace{-\oint_S \overbrace{\rho(\underline{v}\cdot\underline{n})}^{\text{mass flux}}\, da}_{\text{rate @ which mass crosses into boundary.}}$$

Incomp Fluid

$\triangleq (D\rho/Dt) \equiv 0$

$$\boxed{\begin{array}{l}\nabla\cdot\underline{v}=0\\ \text{Cons. of mass for}\\ \text{incomp. fluid.}\end{array}}$$

$$\int_V \left(\frac{\partial\rho}{\partial t} + \nabla\cdot\rho\underline{v}\right) dV = 0$$

arbitrary → 0

$$\boxed{\frac{\partial\rho}{\partial t} + \nabla\cdot\rho v = 0.}$$

$$\left[\frac{\partial\rho}{\partial t} + (v\cdot\nabla)\rho\right] + \rho\,\nabla\cdot v = 0.$$

$$\frac{\partial\rho}{\partial t} + (\underline{v}\cdot\nabla)\rho \equiv \frac{D\rho}{Dt}$$ for observer moving w/ fluid

"material deriv"

"convec. deriv"

(1b) Conservation of Momentum: $\rho \underline{v} \equiv$ momentum density

$\underline{F}$ - force density ; $\underline{f} \equiv$ total force /

$$\underbrace{\frac{d}{dt}\int \rho v_i \, dV}_{\text{rate of accum of momentum}} + \underbrace{\oint_S \overset{\text{momentum flux}}{\rho v_i (\underline{v} \cdot \underline{n})} \, da}_{\text{rate @ which mass crosses boundary}} = \int_V F_i \, dV$$

$$\int \left[\frac{\partial}{\partial t}(\rho v_i) + \nabla \cdot v_i (\rho \underline{v})\right] dV = \int_V F_i \, dV$$

arbitrary $\to 0$

$$\underbrace{v_i \left(\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \underline{v} \right)}_{\text{(cons. of mass)} \; \equiv 0} + \underbrace{\rho \left(\frac{\partial v_i}{\partial t} + (\underline{v} \cdot \nabla) v_i \right)}_{\text{material deriv}} = F_i$$

$$\equiv \underbrace{\rho \frac{D\underline{v}}{Dt}}_{\text{accel.}} = \underbrace{\underline{F}}_{\text{force density.}}$$

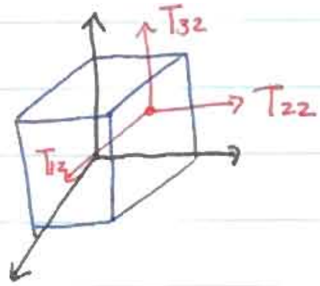
Form of Fluid Flow profile (type of fluid Mech problem that we have) depends on what we include in F.

$$\rho\left(\frac{\partial \underline{v}}{\partial t} + (\underline{v} \cdot \nabla)\underline{v}\right) = \rho \underline{g} - \nabla p + \underbrace{\rho_e E}_{\text{(Lorentz force density)}} + \underbrace{(\overset{\text{dipole moment. density.}}{\underline{P}} \cdot \nabla) E}_{\text{(polarization force density)}}$$

$$+ \underbrace{\text{"}\underline{M} \cdot \underline{\nabla} H\text{"}}_{\text{mag. dipole Force density}} + \underbrace{J \times B}_{\text{Force density from current.}} + \dots + \dots$$

$$+ \underbrace{\mu \nabla^2 \underline{v}}_{\text{viscous force density.}} + \dots + (\nabla \cdot \underline{\underline{T}}^{visc})$$

$T_{ij} \equiv$ Stress Tensor : $\begin{bmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{33} \end{bmatrix}$ $\frac{N}{m^2}$ (Pa)



direc

$i = j$ normal — normal to surface.

$i \neq j$ shear stress.