$\boxed{\text{Today}}$: I. Recap EQS 10/13

Flux, Current, Constitutive Laws

⇒ Coupled Electrochemical Transport

II. Laplace; Poisson: Formulation of Boundary Value Problems & Bound. Conditions for EQS

III. Methods of Solution (Separation of Variables good again)

→ Examples

I Coupled Electrochem Transport

$\left(\frac{mol}{m^3}\right)\left(\frac{m}{s}\right)$

① $\underline{N_i} = -D_i \nabla c_i + (\quad)\underline{E} + (\underset{\substack{\uparrow \\ \text{for units to match}}}{1}) c_i v_{\underline{s}\underline{e}}$

② $\frac{\partial c_i}{\partial t} = -\nabla \cdot \underline{N_i} + R_i$

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③ $\nabla \cdot \varepsilon \underline{E} = \rho_e(\underline{r}, t) = \sum_i z_i F c_i$

valence $\quad 10^5 \frac{\text{coul}}{\text{mol}}$

④ $\nabla \times \underline{E} \simeq 0 \Rightarrow E = -\nabla \Phi$ — Φ = potential (volts)

- because $\nabla \times (-\nabla \Phi) = 0$ for conservative field.
- '−' because Electric field flows down the potential gradient

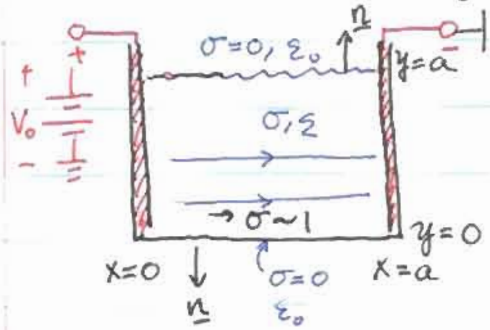
⑤ $\nabla \cdot \underline{J} = -\frac{\partial \rho_e}{\partial t}$

⑥ $\underset{\substack{\uparrow \\ \frac{\text{coul}}{m^2 \cdot s}}}{\underline{J}} = \sum_i z_i F \underset{\substack{\uparrow \\ \frac{\text{mol}}{m^2 \cdot s}}}{\underline{N_i}} = (\text{"}\sigma\text{"})\underline{E} + (\quad)\nabla c_i + (\quad)\underline{v_{\underline{s}\underline{e}}}$

for + charge, + current, moving to the right

− charge, + current, moving to the right

Last Time: Demo



Find: $\Phi(x,y)$ & $\underline{E}(x,y)$ in R given approp. B.C.'s on $\Phi(x,y)$

B.C.'s: $x=0$, $\Phi = V_0$

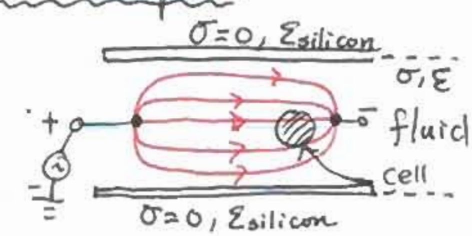
$x=a$, $\Phi=0$

$\left.\begin{array}{l} y=0, \\ y=a \end{array}\right\}$ $n \cdot \nabla\Phi = 0$ ($-E$)

$E_y\left(x, \begin{array}{l} y=0 \\ y=a \end{array}\right) \equiv 0$

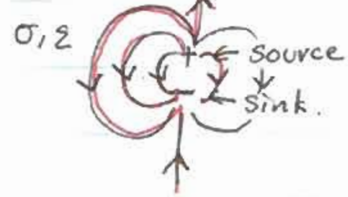
Note: Boundary Conditions on Φ or E

More Examples:



Boundary: between conducting & insulating material.

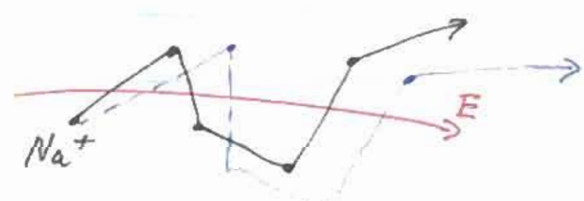
current dipole



Constitutive Law for $\underline{J}$ ($\Rightarrow \underline{N}$!) in Ionic Media (Physiologic)

"ions in thermal equilibrium w/ solvent @ temp T"

"Thought Model"



Einstein Relation

$$\boxed{\frac{D_i}{u_i} = \frac{kT}{q} = \frac{RT}{|z_i|F}}$$

water: $\left(\frac{1000\ gm}{l}\right)\left(\frac{mol}{18gm}\right) \sim 55\ M$

$$\cancel{\frac{m\,dv}{dt}}^{\,0} + \frac{mv}{\tau_{col}} = q\underline{E} \rightarrow \underline{v}_{ion\,\pm} = \pm \underbrace{\left(\frac{|q|\tau}{m}\right)}_{u_i \equiv \text{mobility of species}} \underline{E}$$

$\left(\frac{m^2}{v \cdot s}\right)$ $u_i \equiv$ mobility of species

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$\underline{J} = \sum z_i F(c_i v_{particle})$

$$\underline{J} = \underbrace{F(z_+ c_+ u_+ + z_- c_- u_-)}_{\text{"ohmic"}} E + \underbrace{(\ )\nabla c}_{\text{diff.}} + \text{convection.}$$

σ. elec cond. $\left(\frac{mho}{m}\right) \equiv \left(\frac{S}{m}\right)$

Note: charge not changing the E_{total}.

II. Poisson / Laplace (• No ∇C_i; $v_{fl} = 0$) • $L^{char} \ll \lambda$

"Static" (steady cardiac) vs time varying - non equal)

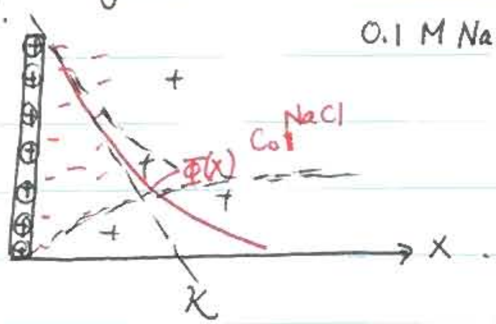
?? Maxwell	EQS.
$\nabla \cdot \varepsilon E = \rho$	$\nabla \cdot \varepsilon E = \rho$
$\nabla \times E = -\frac{\partial \mu H}{\partial t}$	$\nabla \times E = 0$ $(E = -\nabla \Phi)$
$\nabla \times H = J + \frac{\partial}{\partial t} \varepsilon E$	
$\nabla \cdot \mu H = 0$	
$\nabla \cdot J = -\frac{\partial \rho}{\partial t}$.	$\nabla \cdot J = -\frac{\partial \rho_e}{\partial t}$
	$\underline{J} = \sigma \underline{E}$

From EQS In general: ① + ② $\nabla \cdot \varepsilon(-\nabla \Phi) = \rho_e(r,t)$ Gauss.

$$\boxed{\nabla^2 \Phi = -\frac{\rho_e}{\varepsilon}}$$ Poisson's Eq.

Find $\Phi \rightarrow$ get $E = -\nabla \Phi$.

Ex. 4.1.



$\frac{1}{\kappa} \sim$ Debye Length

$\downarrow$

Find $\Phi(x)$ given $C_+(x)$

$C_+(x) = C_0 e^{-zF\Phi/RT}$

Class of Problems: Steady Conduction $\left(\frac{\partial}{\partial t} \rightarrow 0\right)$ "static"

$\nabla \cdot \varepsilon \underline{E} = \rho_e$ (ohmic medium)

$\checkmark\ \underline{E} = -\nabla \Phi$

$\checkmark\ \nabla \cdot \sigma E = 0$

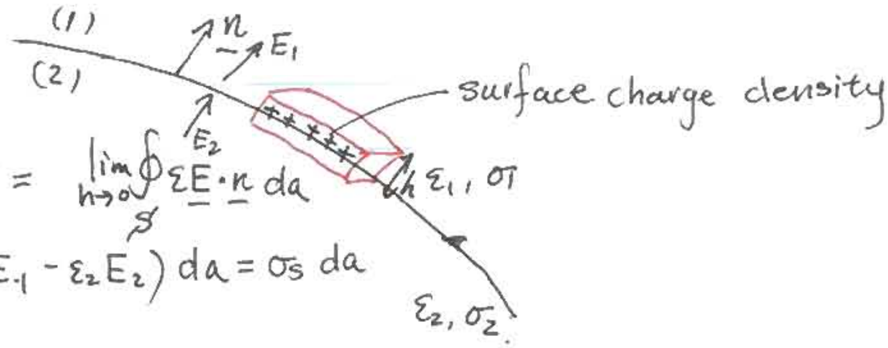
$\boxed{\nabla^2 \Phi = 0}$ Laplace's Equation

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} = 0.$$

Find $\Phi(r, t)$?? in region R given boundary conditions on Φ or $\underline{n} \cdot \nabla \Phi$ on S surrounding R.

Remember B.C's.

Law	Boundary Conditions
$\nabla \cdot \varepsilon E = \rho_e$	$\underline{n} \cdot (\varepsilon_1 E_1 - \varepsilon_2 E_2) = \sigma_s$
$\nabla \times E = 0$	E_{tan} = continuous $\Phi_1 = \Phi_2$ @ interface.
$\nabla \cdot J = -\dfrac{\partial \rho_e}{\partial t}$	$n \cdot (\sigma_1 E_1 - \sigma_2 E_2) = -\dfrac{\partial}{\partial t} \sigma_s$



$$\int_V \rho_e \, dV = \lim_{h \to 0} \oint_S \varepsilon \underline{E} \cdot \underline{n} \, da$$

$$n \cdot (\varepsilon_1 E_1 - \varepsilon_2 E_2) \, da = \sigma_s \, da$$