
BEH.430/2.795//6.561/10.539/HST.544

Homework Set 2

Handed out: Wednesday Sept. 22, 2004

Due: Wednesday, Sept 29 by 5pm

Problem 2.1: Transient Diffusion across Tissue Membrane (No Reaction)

(a) This problem focuses on the use of Separation of Variables to solve for the evolution of the space-time concentration profile of IGF-1, $c(x,t)$, inside a section of tissue having length L (i.e., 1-D profile). As stated in class, the boundary conditions are $c(0,t) = c'$; $c(L,t) = c''$; and the initial condition is $c(x, 0) = 0$. You are asked to carry through the details of the solution, and to derive in detail the coefficients of all the terms in the Fourier series, showing all the steps that yield the final analytical expression for $c(x,t)$ written in lecture and on the overhead (posted with the lecture overheads). You can assume in part (a) that the partition coefficient valid at $x = 0$ and $x = L$ is $K = 1$.

(b) Suppose $K \neq 1$ (e.g., for the case in class, $K = 1.4$). Will this affect the kinetics of the diffusion transient? State your reasoning. Write the answer given in (a) above but now including the non-unity partition coefficient.

Problem 2.2: Time Lag Measurement of the Diffusion Coefficient: An extremely important and commonly used experimental technique

(a) Based on your answer to Problem 2.1, part (a) above, find an analytical expression for the flux of IGF evaluated at the right hand edge of the tissue, $x = L$, at any time t throughout the transient diffusion process across the tissue. By integrating the flux with respect to time, t , find the total amount of IGF that has moved across the tissue into the right hand chamber in a time, t . (If you divide by the volume, V , of the right hand chamber, you'll have an expression for the concentration $c''(t)$.)

(b) As $t \rightarrow \infty$, the graph of $c''(t)$ versus time, t , approaches a straight line as a function of t . Find the extrapolated intercept of the line on the t -axis, and show that this time intercept is related to the solute diffusivity, D , by the value $L^2/(6D)$ as stated in class.

Problem 2.3: The time constant for equilibration of $c'(t)$ and $c''(t)$ across the tissue.

In Problems 2.1 and 2.2 above, we have assumed that c' and c'' are constants independent of time. In reality, if we wait long enough, the concentrations will equilibrate such that $c' = c'' = (c'/2)$ assuming that $c'' = 0$ at $t = 0$. This problem asks you to find an expression for the "chamber equilibration" time constant.

(a) Assume that c' and c'' are uniform within each respective bath compartment for all time (i.e., the compartments are well stirred). Assume that sufficient time has passed that a linear concentration profile of c exists within the tissue for all times of interest during the overall slow equilibration process. Find an expression for the solute flux across the tissue in terms of c' , c'' , L , and D . (Assume that partition coefficient $K = 1$; assume that right hand and left hand chambers both have volume V).

(b) Using conservation of mass ($[Vc'(t) + Vc''(t)] = \text{constant} = V\beta$), derive a differential equation for the time rate of change of the uniform bath concentration $c'(t)$ as solute moves from left to right across the tissue.

(c) Obtain the solution to the differential equation (should be a simple exponential decay), and show that your solution satisfies the expected infinite time response, $c'(t = \infty) = \beta/2$. Compare your exponential decay time constant to the characteristic diffusion time for a solute moving across a length, L , $\tau_{\text{diff}} = L^2/(\pi^2 D)$, and clarify the validity of the assumption of the “steady state” linear concentration profile in Prob. 2.1.