Lecture 18: Fields and Moving Media

November 15,2005

I. Ohm's Law in Moving Media

Force on moving charge: $\overline{f} = q\left(\overline{E} + \overline{v} \times \overline{B}\right)$

Consider force on moving charge in reference frame (denoted as prime (') frame) moving at charge velocity $\overline{v}$. Then $\overline{f}' = q\overline{E}'$ as in the moving frame the relative velocity is zero.

With $\overline{v}$ constant, $\overline{f}$ and $\overline{f}'$ are inertial frames (non-accelerating) so that:

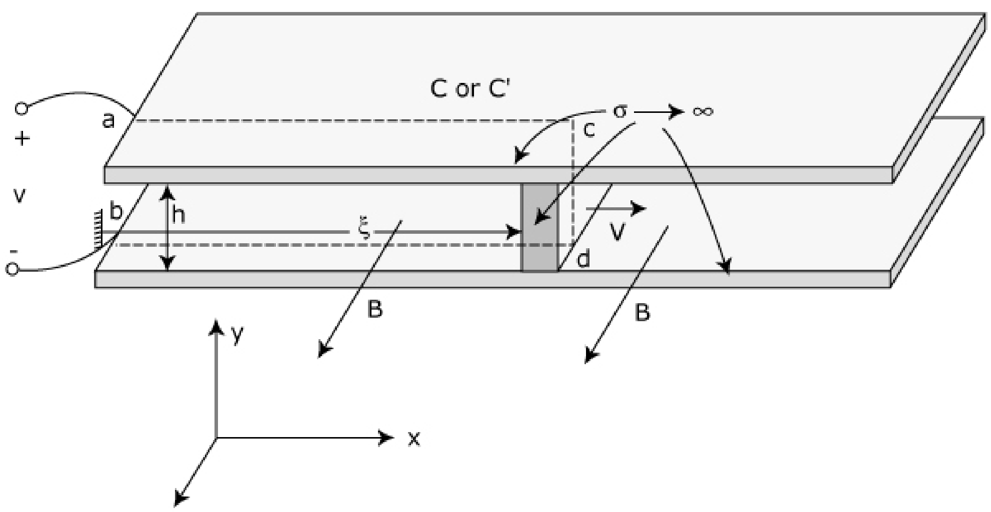
$$\overline{f} = \overline{f}' \Rightarrow \cancel{q}\overline{E}' = \cancel{q}\left(\overline{E} + \overline{v} + \overline{B}\right) \Rightarrow \overline{E}' = \overline{E} + \overline{v} + \overline{B}$$

in primed frame: $\overline{J}' = \sigma\overline{E}' = \sigma\left(\overline{E} + \overline{v} \times \overline{B}\right)$

If system is charge neutral, as is usual case in MQS systems

$$\overline{J} = \overline{J}' = \sigma\left(\overline{E} + \overline{v} \times \overline{B}\right)$$

II. Moving Media MQS Problem



A pair of parallel perfectly conducting plates are short-circuited by a moving perfectly conducting bar. Because of the magnetic field B, a voltage v is induced which can be computed either by integrating the induction equation around the fixed loop C' that passes through the bar or by integrating the induction equation around a loop C that expands in area as the bar moves to the right. The field transformation of (6.1.38) guarantees that both integrations will give the same result.

Moving Contour C

$$\oint_C \overline{E}' \bullet \overline{dl} = -\frac{d}{dt}\int_S \overline{B} \bullet \overline{n}\, da$$

$$-v = -\frac{d}{dt}\left[-B\, h\, \xi\right]$$

$\overline{B}$ and $\overline{n}$ in opposite directions

$$v = -B\, h \frac{d\xi}{dt} = -B\, h\, V$$

Stationary Contour C'

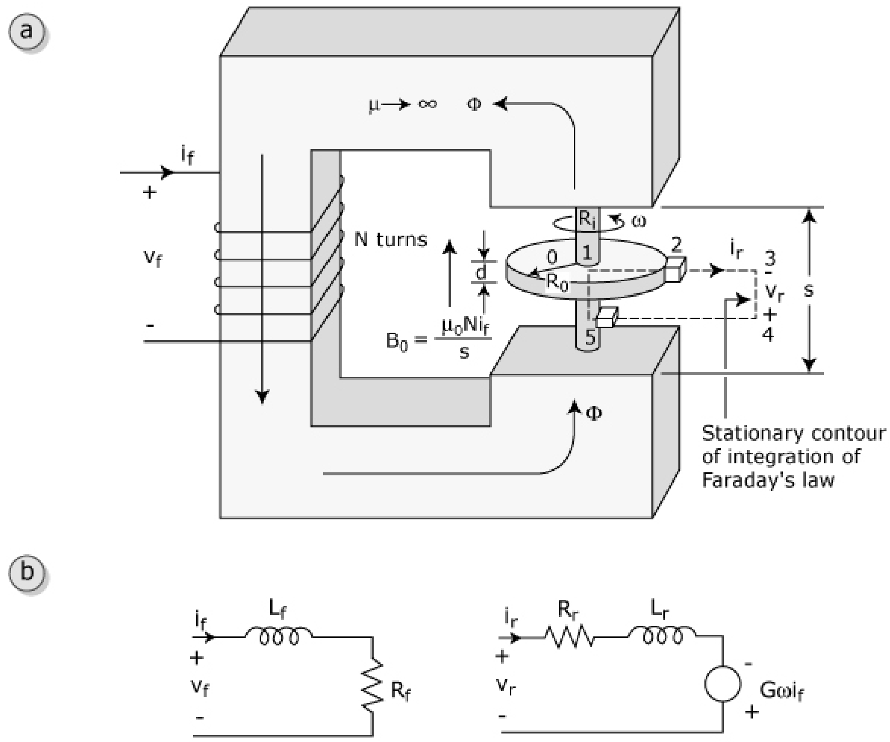
$$\oint_{C'} \overline{E} \bullet \overline{dl} = -\frac{d}{dt}\int_{S'} \overline{B} \bullet \overline{n}\, da = 0$$

$\overline{E}' = 0 = \overline{E} + \overline{v} \times \overline{B}$ in moving perfect conductor

$$-v + \left(\overline{v} \times \overline{B}\right)_y h = 0$$

$$v = -B\, h\, V$$

III. Faraday's Disk (Homopolar Generator)



(a) A conducting disk rotating in an axial magnetic field is called a homopolar generator. (b) In addition to Ohmic and inductive voltages there is a speed voltage contribution proportional to the speed of the disk and the magnetic field.

$$B_0 = \frac{\mu_0 N i_f}{s}$$

$$\bar{J} = \sigma\left(\bar{E} + \bar{v} \times \bar{B}\right) \Rightarrow \bar{E} = \frac{\bar{J}}{\sigma} - \bar{v} \times \bar{B} \Rightarrow E_r = \frac{i_r}{2\pi\sigma d r} - \omega r B_0$$

$$\oint_L \bar{E} \bullet \overline{dl} = \int_1^2 E_r \, dr + \underbrace{\int_3^4 \bar{E} \bullet \overline{dl}}_{-v_r} = 0$$

$$v_r = \int_1^2 E_r \, dr = \int_{R_i}^{R_0} \left(\frac{i_r}{2\pi\sigma d r} - \omega r B_0\right) dr = \frac{i_r}{2\pi\sigma d} \ln\frac{R_0}{R_i} \frac{-\omega B_0}{2}\left(R_0^2 - R_i^2\right)$$

$$= i_r R_r - G\,\omega\, i_f$$

$$R_r = \frac{\ln \frac{R_0}{R_i}}{2\pi\sigma d}\ , \quad G = \frac{\mu_0 N}{2s}\left(R_0^2 - R_i^2\right)$$

Representative Numbers: copper $\left(\sigma \approx 6 \times 10^7 \text{ siemen/m}\right)$, $d = 1\,\text{mm}$

$\omega = 3600\,\text{rpm} = 120\,\pi\,\text{rad/s}$

$R_0 = 10\,\text{cm},\ R_i = 1\,\text{cm},\ B_0 = 1\,\text{tesla}$

$$v_{0c} = \frac{-\omega B_0}{2}\left(R_0^{\,2} - R_i^{\,2}\right) \approx -1.9\ \text{V}$$

$$i_{sc} = \frac{v_{0c}\, 2\pi\sigma d}{\ln\left(\frac{R_0}{R_i}\right)} \approx 3 \times 10^5\ \text{amp}$$

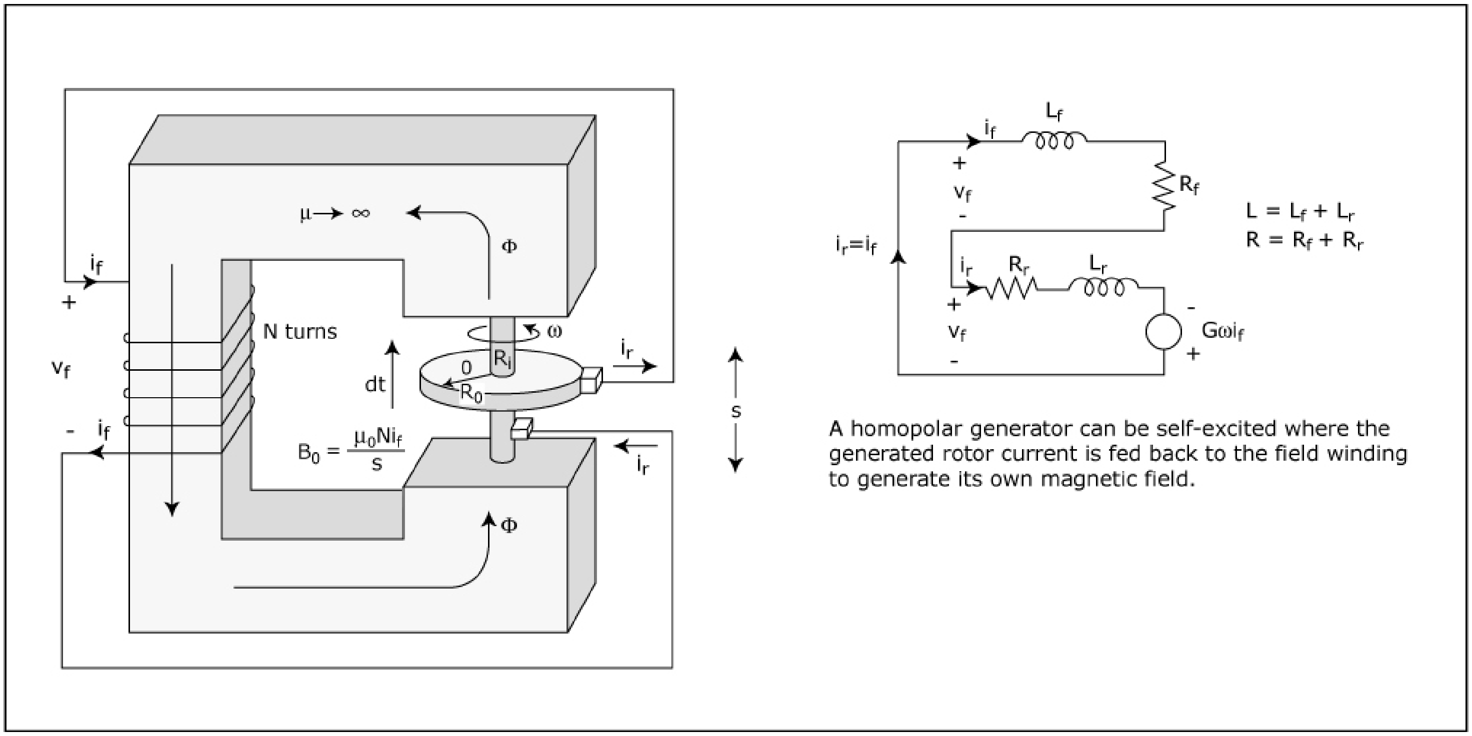
$$\bar{T} = \int_{\phi=0}^{2\pi}\int_{z=0}^{d}\int_{r=R_i}^{R_0} r\,\bar{i}_r \times \left(\bar{J} \times \bar{B}\right) r\,dr\,d\phi\,dz$$

$$= -i_r B_0\,\bar{i}_z \int_{R_i}^{R_0} r\,dr$$

$$= \frac{-i_r B_0}{2}\left(R_0^{\,2} - R_i^{\,2}\right)\bar{i}_z$$

$$= -G\, i_f\, i_r\, \bar{i}_z$$

IV. Self-Excited DC Homopolar Generator



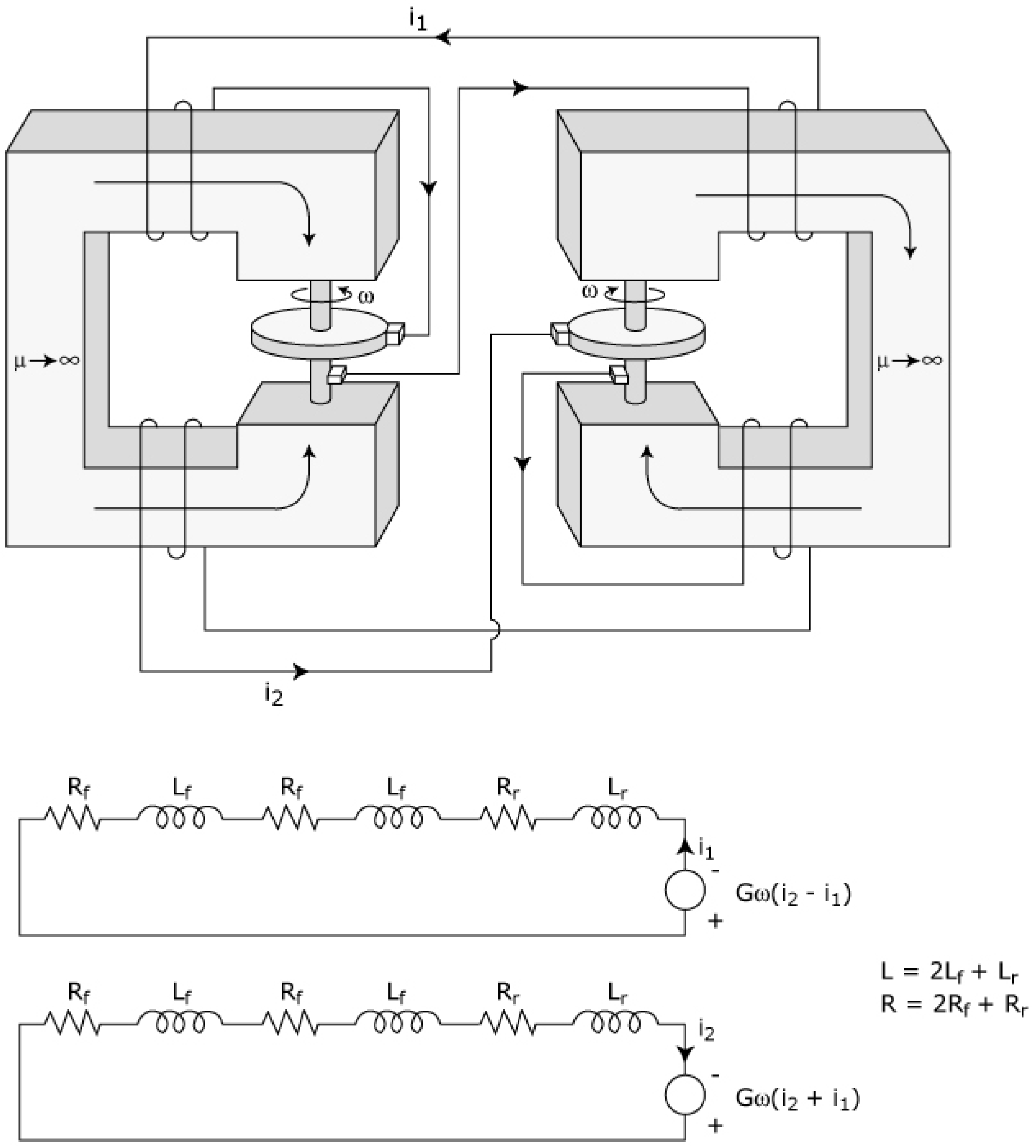
$$i_f = i_r \equiv i$$

$$L\frac{di}{dt} + i(R - G\omega) = 0 \;; \quad R = R_r + R_f$$
$$L = L_r + L_f$$

$$i(t) = I_0 e^{-[R-G\omega]t/L}$$

$G\omega > R$ Self-Excited

V. Self-Excited AC Homopolar Generator



Cross-connecting two homopolar generators can result in self-excited two-phase alternating currents. Two independent field windings are required where on one machine the fluxes add while on the other they subtract.

$$L\frac{di_1}{dt} + (R - G\omega)i_1 + G\omega i_2 = 0$$

$$L\frac{di_2}{dt} + (R - G\omega)i_2 - G\omega i_1 = 0$$

$$i_1 = I_1 e^{st}, \quad i_2 = I_2 e^{st}$$

$$\left(Ls + R - G\omega\right) I_1 + G\omega I_2 = 0$$

$$-G\omega I_1 + \left(Ls + R - G\omega\right) I_2 = 0$$

$$\left(Ls + R - G\omega\right)^2 + \left(G\omega\right)^2 = 0$$

$$Ls + R - G\omega = \pm jG\omega$$

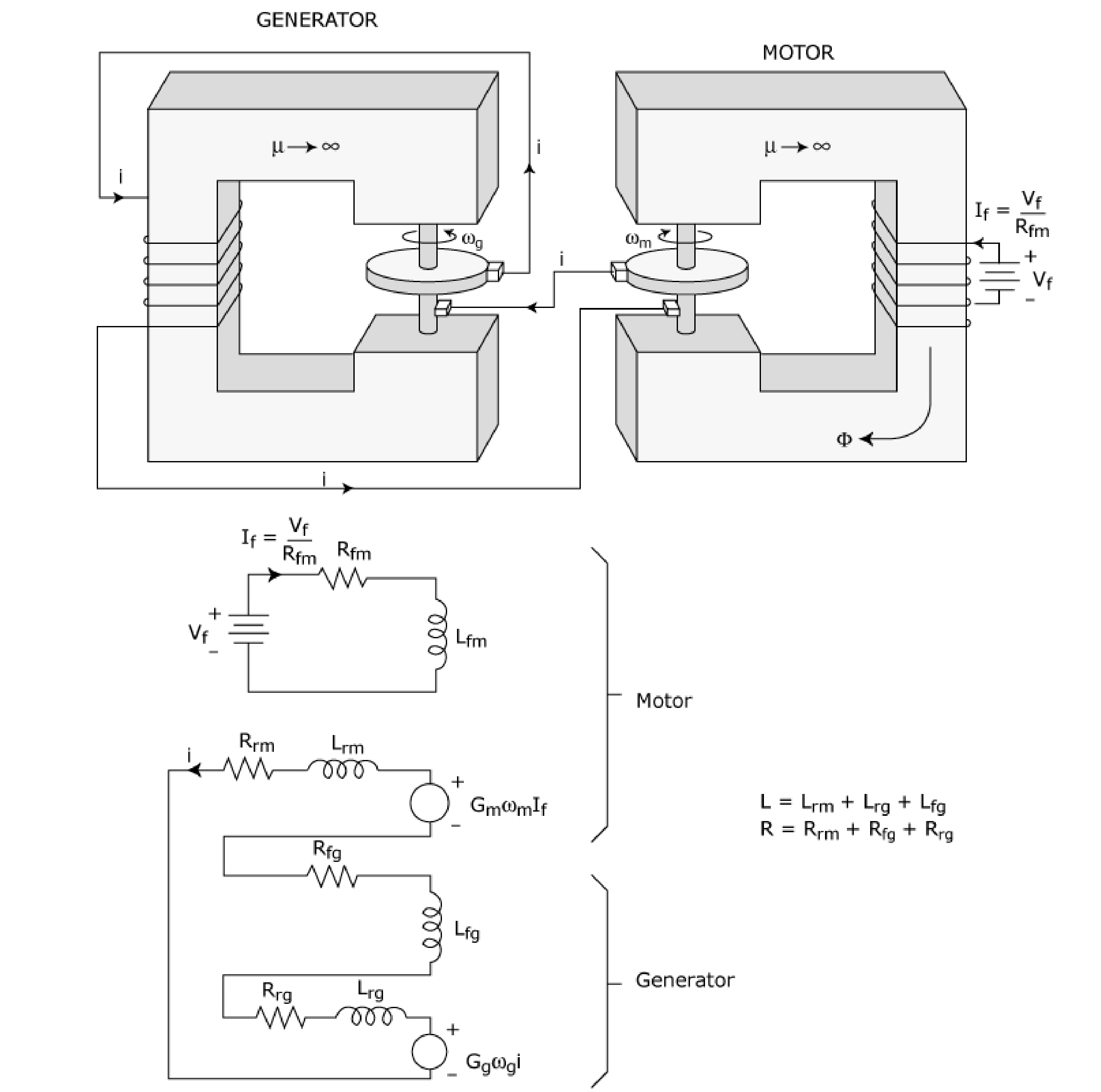
$$s = -\frac{\left(R - G\omega\right)}{L} \pm j\frac{G\omega}{L}$$

$$\frac{I_1}{I_2} = \frac{-G\omega}{\left(Ls + R - G\omega\right)} = \pm j$$

Self Excited: $G\omega > R$

Oscillation frequency: $\omega_0 = I_m\left(s\right) = G\omega/L$

VI. Self-Excited Periodic Motor Speed Reversals



Cross-connecting a homopolar generator and motor can result in spontaneous periodic speed reversals of the motor's shaft.

$$\frac{di}{dt} + \frac{\left(R - G_g\,\omega_g\right) i}{L} = \frac{G_m\,\omega_m\,I_f}{L}$$

$$J \frac{d\omega_m}{dt} = -G_m I_f i$$

$$i = I e^{st}, \omega_m = W e^{st}$$

$$I \left[s + \frac{R - G_g \omega_g}{L} \right] - W \left(\frac{G_m I_f}{L} \right) = 0$$

$$I \left(\frac{G_m I_f}{J} \right) + Ws = 0$$

$$s \left[s + \frac{R - G_g \omega_g}{L} \right] + \frac{(G_m I_f)^2}{JL} = 0$$

$$s = -\frac{(R - G_g \omega_g)}{2L} \pm \left[\left(\frac{R - G_g \omega_g}{2L} \right)^2 - \frac{(G_m I_f)^2}{JL} \right]^{1/2}$$

Self-excitation: $G_g \omega_g > R$

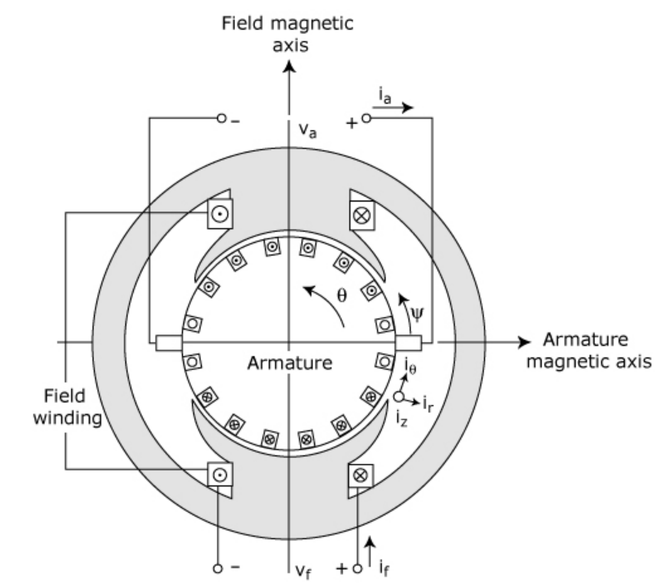
Oscillations if s has an imaginary part:

$$\frac{(G_m I_f)^2}{JL} > \left(\frac{R - G_g \omega_g}{2L} \right)^2$$

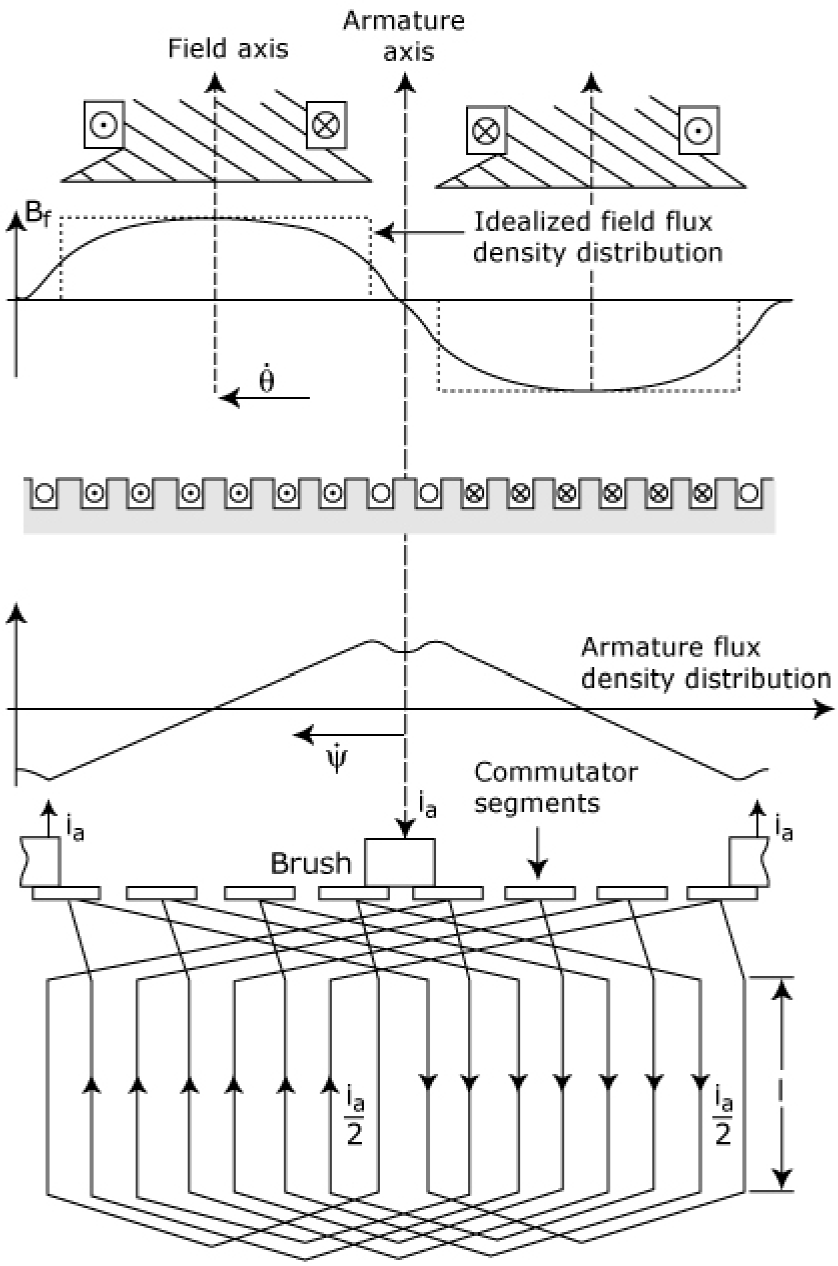
VII. DC Commutator Machines

Quasi-One Dimensional Description

A. Electrical Equations



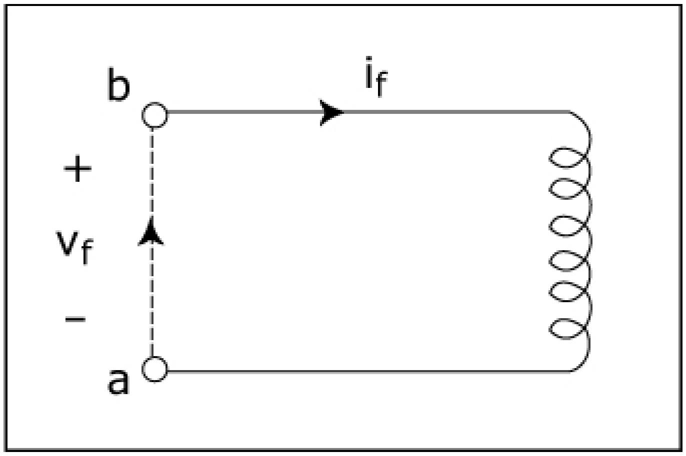
Two-pole, commutator machine.



Developed views of two-pole commutator machine showing flux distributions and armature connections.

$$\oint_C \bar{E} \cdot \overline{dl} = -\frac{d}{dt}\int_S \bar{B} \cdot \bar{n}\, da$$

1. Field Winding



$$\oint_C \bar{E} \cdot \overline{dl} = -v_f + \underbrace{\int_{winding} \frac{i_f}{A\sigma}}_{\frac{J}{\sigma}} dl = -v_f + i_f \underbrace{R_f}_{\text{Resistance of field winding}}$$

$$\lambda_f = \int_S \bar{B} \cdot \bar{n}\, da = L_f\, i_f$$

$$-v_f + i_f R_f = -L_f \frac{di_f}{dt}$$

$$v_f = L_f \frac{di_f}{dt} + i_f R_f$$

2. Armature Winding

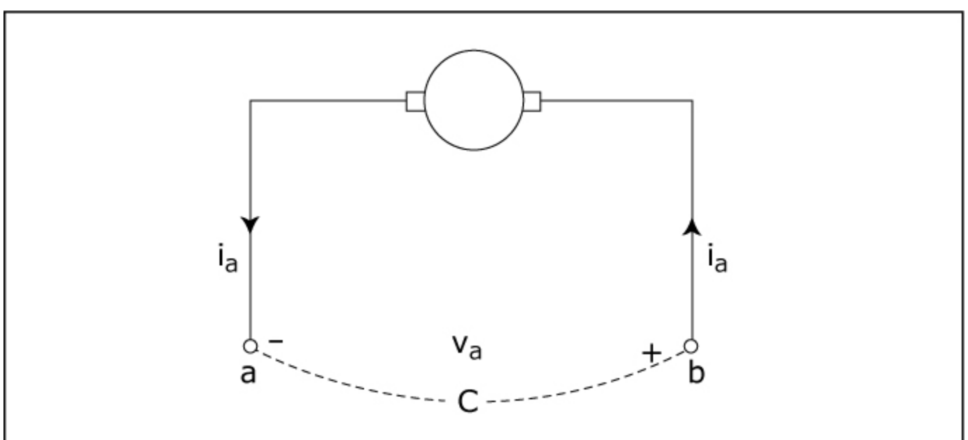


Illustration of the contour for finding an armature voltage equation. The contour is completed on the armature, where it follows one of the two conducting paths joining the brushes in figure on page 16.

Reminder: $\bar{f} = q\left(\bar{E} + \bar{v} \times \bar{B}\right) = q\bar{E}'$

$$\bar{E}' = \bar{E} + \bar{v} \times \bar{B}$$

Take Stationary Contour through armature winding

$$\bar{E} = \bar{E}' - \bar{v} \times \bar{B}$$

$$\oint_C \bar{E} \cdot \overline{dl} = -v_a + \int \left(\bar{E}' - \bar{v} \times \bar{B}\right) \cdot \overline{dl}$$

$$= -v_a + \int_a^b \left(\frac{i_a}{A\sigma} + \omega R B_r \, \bar{i}_z\right) \cdot dl \quad ; \qquad v = \omega R \, \bar{i}_\theta$$

$$\bar{B} = i_r B_r(\chi)$$

$$= -v_a + i_a R_a + \omega R \left(B_{rf}\right)_{av} l N$$

$$= -\frac{d}{dt} \int_S \bar{B} \cdot \overline{da} = -L_a \frac{di_a}{dt}$$

$$v_a = i_a R_a + L_a \frac{di_a}{dt} + G \omega i_f \qquad \left(G i_f = l N R \left(B_{rf}\right)_{av}\right)$$

B. Mechanical Equations

$$\bar{F} = \bar{i}_\theta J_z B_r = \bar{i}_\theta \frac{i_a}{A_w} B_r, \; \bar{f} = \bar{F} A_w l = \bar{i}_\theta i_a l B_r$$

$$T = f R = i_a l B_r R N = G i_f i_a$$

$$J \frac{d^2\theta}{dt^2} = T = G i_f i_a$$

C. Linear Amplifier

1) Open Circuit

$$v_f = V_f, \; i_a = 0 \Rightarrow i_f = V_f / R_f$$

$$v_a = G \omega V_f / R_f$$

2) Resistively Loaded Armature (DC Generator)

$$v_a = -i_a R_L = i_a R_a + G \omega V_f / R_f$$

$$i_a = \frac{-G\,\omega\,V_f}{R_f\,(R_a + R_L)}$$

$$v_a = \frac{G\,\omega\,V_f\,R_L}{R_f\,(R_a + R_L)}$$

D. DC Motors

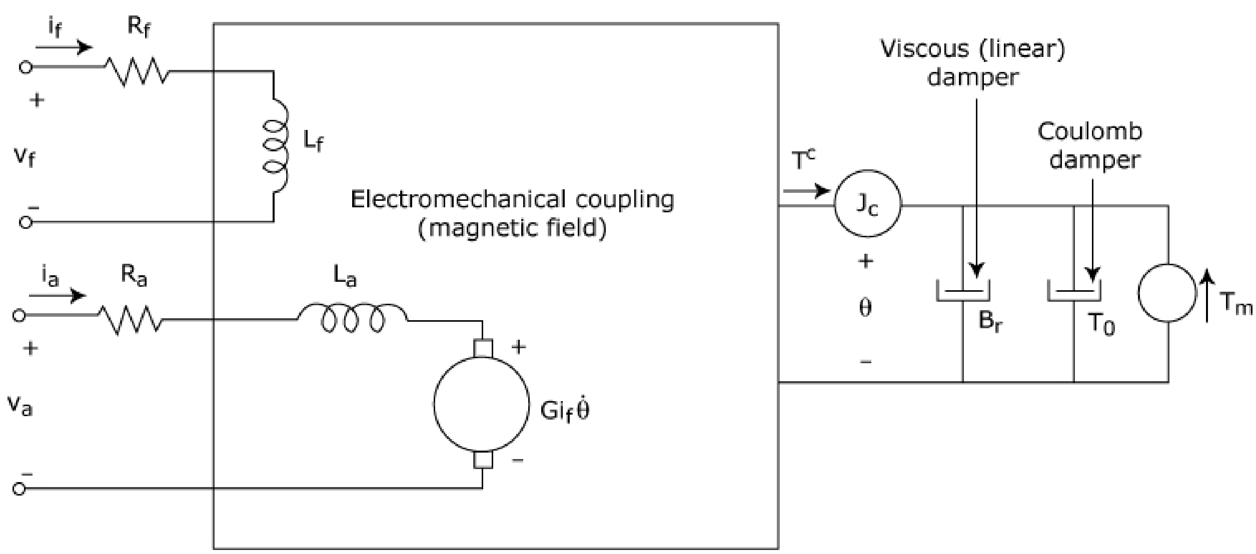
1) Shunt Excitation: $v_a = v_f = v_t$

$$v_t = i_f R_f = i_a R_a + G\,\omega\,i_f$$

$$i_f\,(R_f - G\omega) = i_a R_a$$

$$i_f = \frac{V_t}{R_f},\ i_a = \frac{V_t}{R_f}\frac{(R_f - G\omega)}{R_a}$$

$$T = G i_f\, i_a = G\left(\frac{V_t}{R_f}\right)^2 \frac{(R_f - G\omega)}{R_a}$$



Equivalent circuit of the commutator machine in figure on page 15.

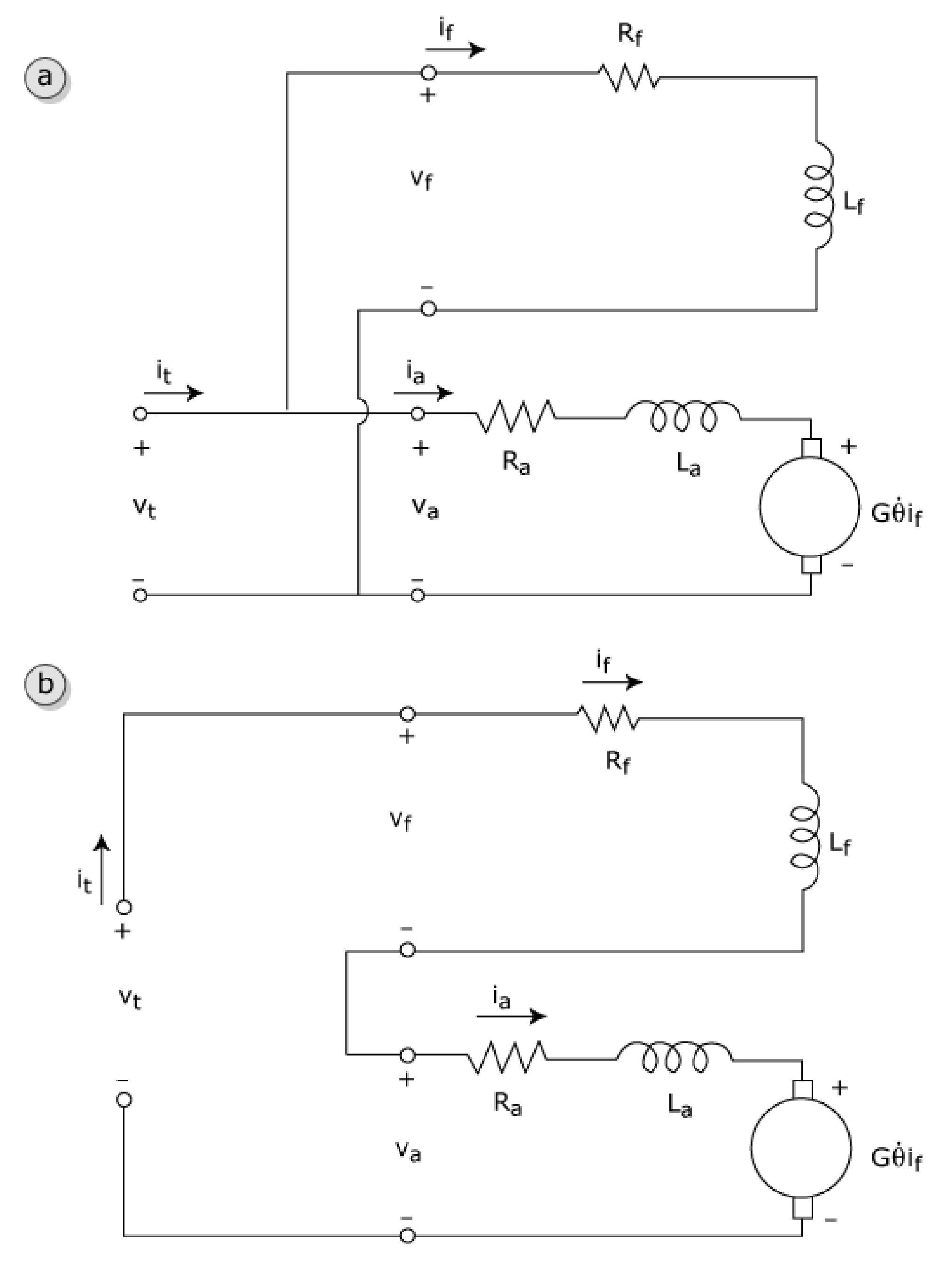
2) Series: $i_a = i_f = i_t$

$$i_t\left(R_f + R_a + G\omega\right) = v_t$$

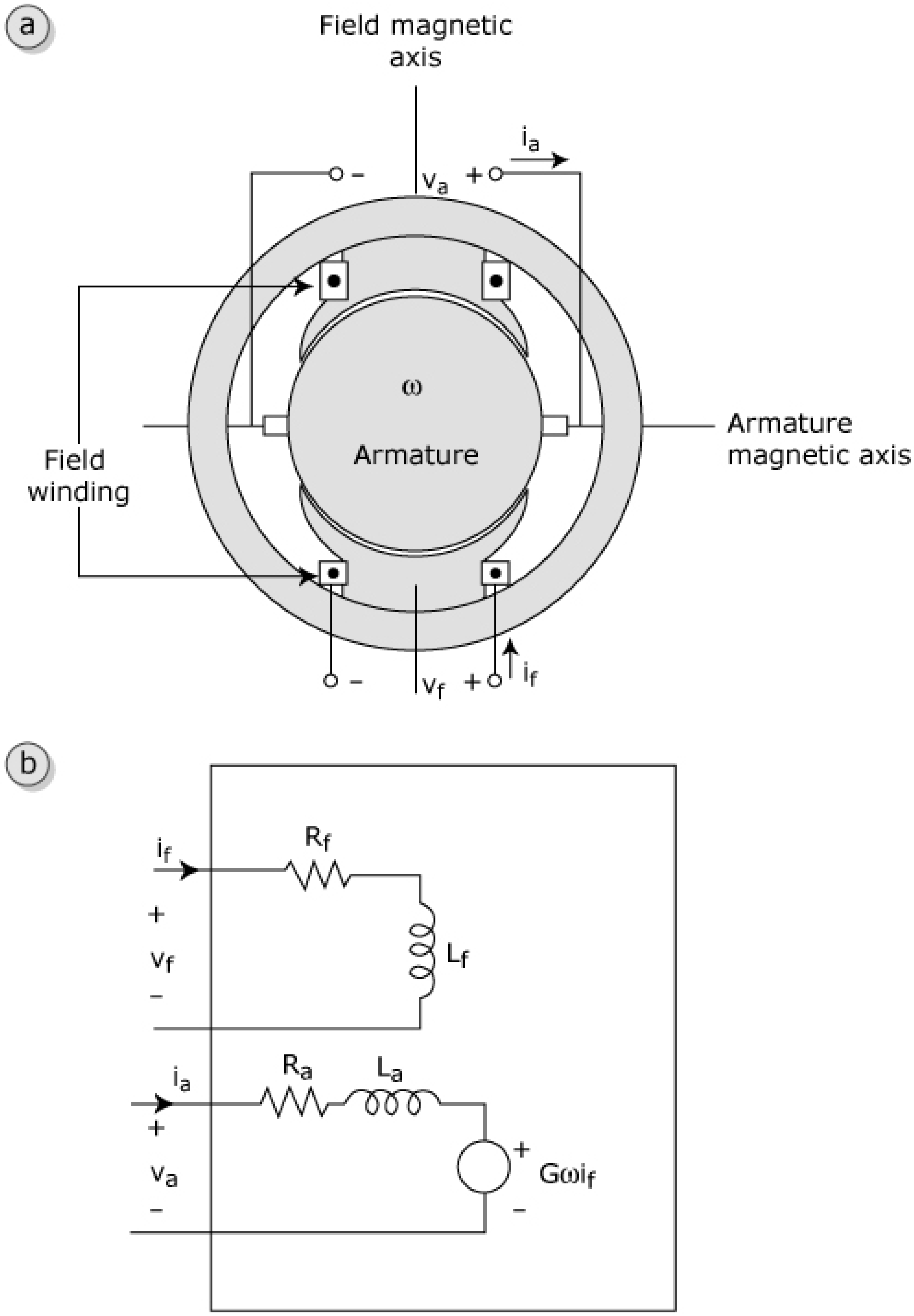
$$i_t = \frac{v_t}{\left(R_f + R_a + G\omega\right)}$$

$$T = G i_t^2 = G\frac{v_t^2}{\left(R_f + R_a + G\omega\right)^2}$$

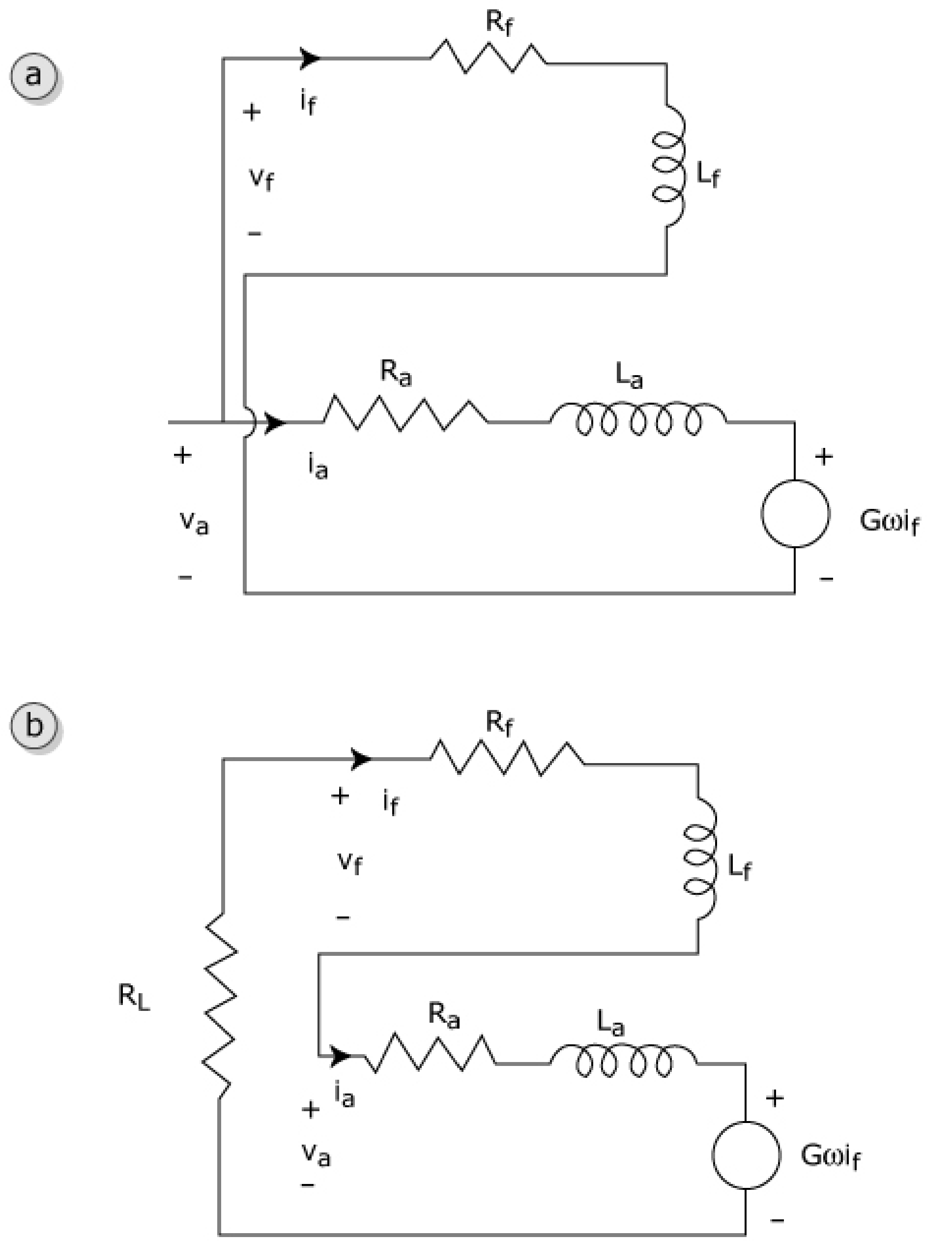
VIII. Self-Excited Machines



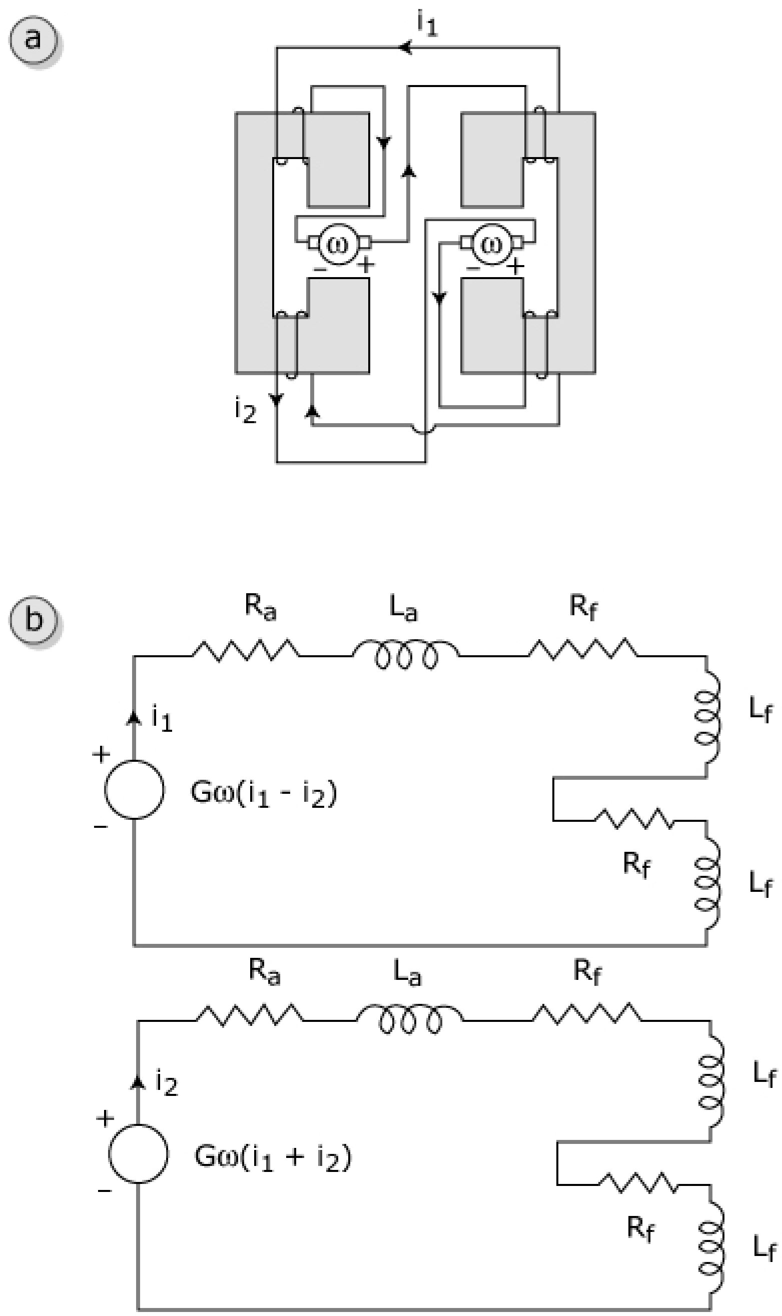
Methods of self-exciting a dc motor: (a) shunt excitation; (b) series excitation.



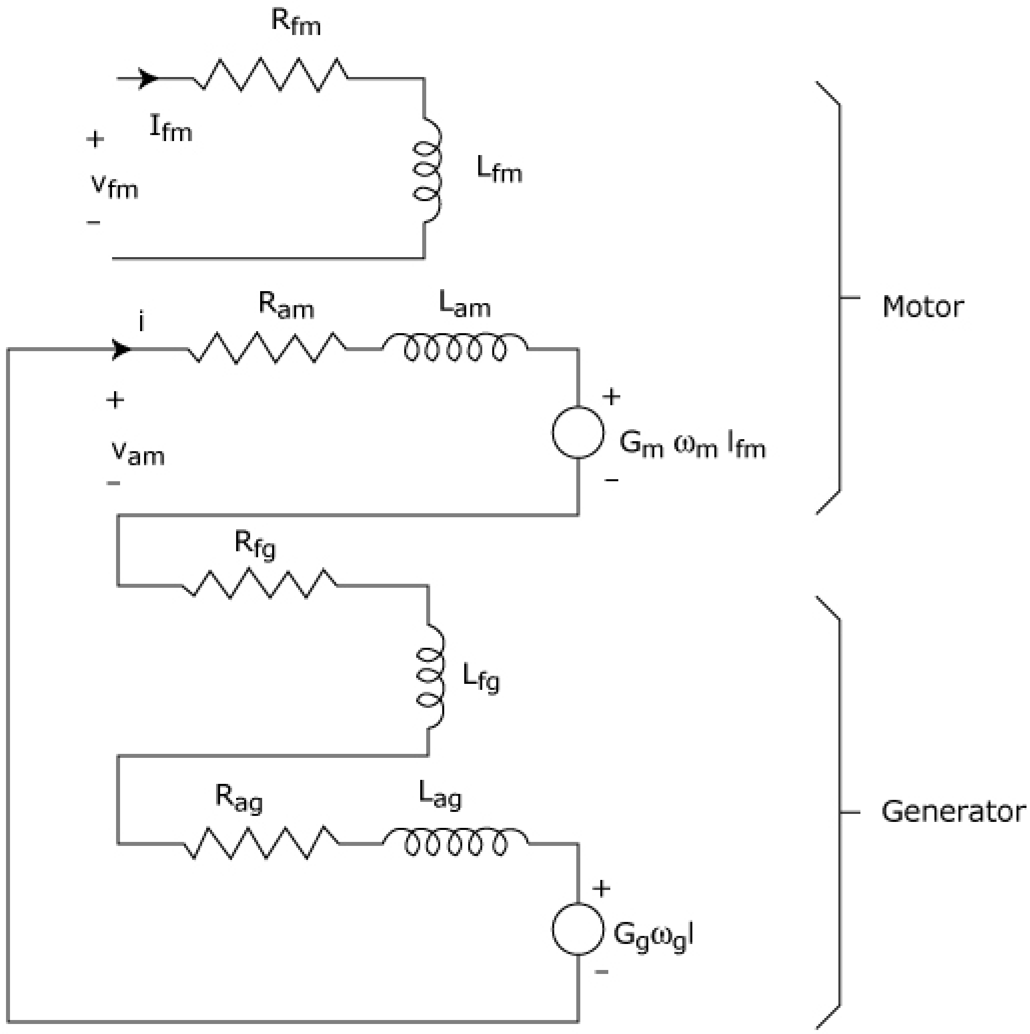
Two-pole commutator machine. (a) Geometry. (b) Equivalent circuit, as derived by Woodson and Melcher (1).



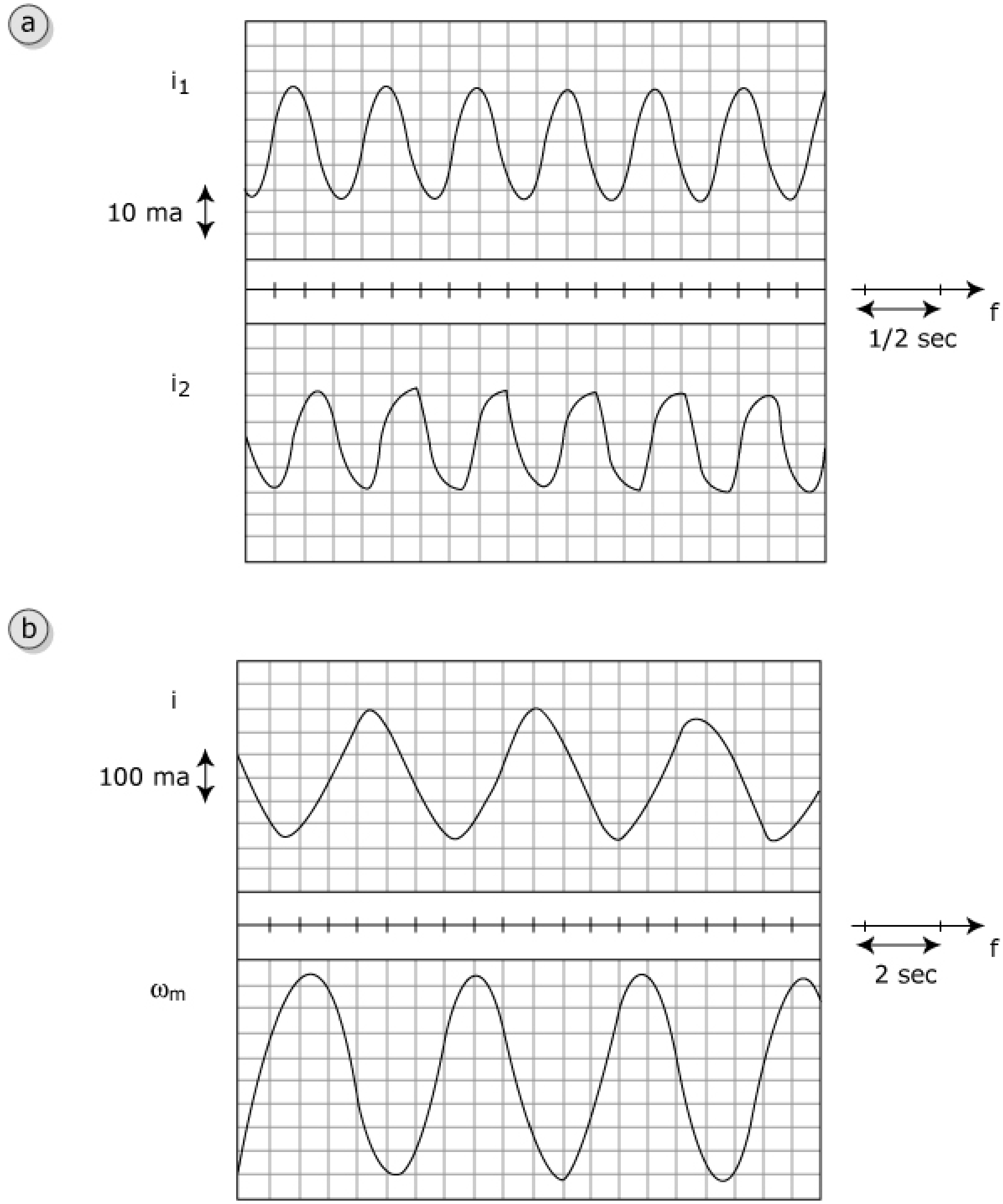
Equivalent circuit models for shunt and series self-excited generators. (a) Open-circuit shunt. (b) Series with load resistor.



Configuration for obtaining a.c. power from two identical generators.
(a) Wiring configuration. (b) Equivalent circuit description.



Equivalent circuit description to obtain spontaneous electrical and mechanical oscillations.



(a) Two-phase currents obtained from a pair of coupled d.c. machines rotating at a speed of 1790 rev/min. (b) Alternating current and speed for the coupled motor-generator combination with If = 0.15 A and generator shaft of 1620 rev/min.