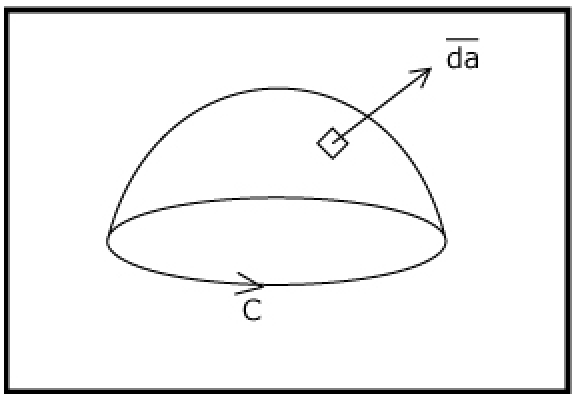
Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
6.013 Electromagnetics and Applications
Lecture 1, Sept. 8, 2005

I. Maxwell's Equations in Integral Form in Free Space

1. Faraday's Law

$$\underbrace{\oint_C \bar{E} \cdot \overline{ds}}_{\substack{\text{Circulation} \\ \text{of } \bar{E}}} = -\frac{d}{dt} \underbrace{\int_S \mu_0 \bar{H} \cdot \overline{da}}_{\text{Magnetic Flux}}$$



$\mu_0 = 4\pi \times 10^{-7}$ henries/meter [magnetic permeability of free space]

EQS form: $\oint_C \bar{E} \cdot \overline{ds} = 0$ (Kirchoff's Voltage Law, conservative electric field)

MQS circuit form: $v = L\frac{di}{dt}$ (Inductor)

2. Ampère's Law (with displacement current)

$$\underbrace{\oint_C \bar{H} \cdot \overline{ds}}_{\substack{\text{Circulation} \\ \text{of } \bar{H}}} = \underbrace{\int_S \bar{J} \cdot \overline{da}}_{\substack{\text{Conduction} \\ \text{Current}}} + \underbrace{\frac{d}{dt} \int_S \varepsilon_0 \bar{E} \cdot \overline{da}}_{\substack{\text{Displacement} \\ \text{Current}}}$$

MQS form: $\oint_C \bar{H} \cdot \overline{ds} = \int_S \bar{J} \cdot \overline{da}$

EQS circuit form: $i = C\frac{dv}{dt}$ (capacitor)

3. Gauss' Law for Electric Field

$$\oint_S \varepsilon_0 \overline{E} \cdot \overline{da} = \int_V \rho \, dV$$

$$\varepsilon_0 \approx \frac{10^{-9}}{36\pi} \approx 8.854 \times 10^{-12} \text{ farads/meter}$$

$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \approx 3 \times 10^8$ meters/second (Speed of electromagnetic waves in free space)

4. Gauss' Law for Magnetic Field

$$\oint_S \mu_0 \overline{H} \cdot \overline{da} = 0$$

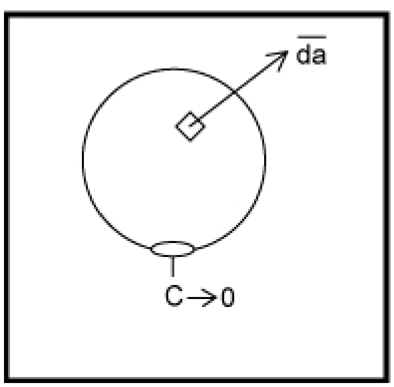
In free space:

$$\overline{B} = \mu_0 \overline{H}$$

$\overline{B}$	$\overline{H}$
magnetic flux density (Teslas)	magnetic field intensity (amperes/meter)

5. Conservation of Charge

Take Ampère's Law with displacement current and let contour $C \rightarrow 0$



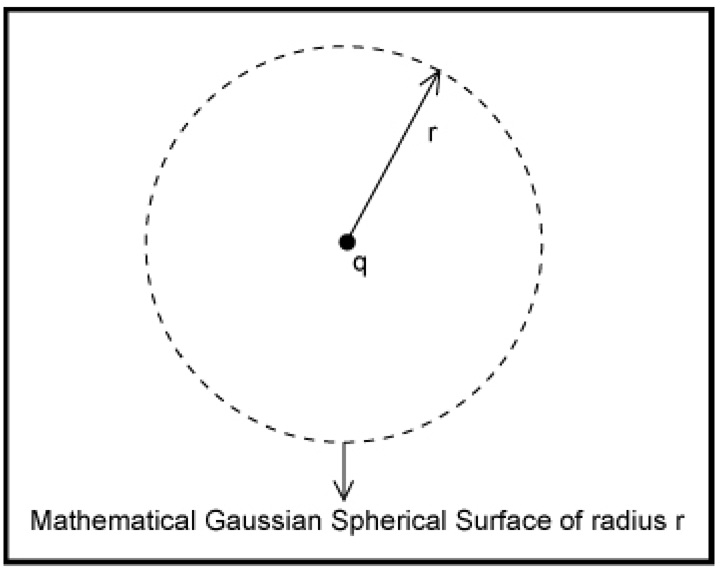
$$\lim_{C \to 0} \oint_C \overline{H} \cdot \overline{ds} = 0 = \oint_S \overline{J} \cdot \overline{da} + \frac{d}{dt} \underbrace{\oint_S \varepsilon_0 \overline{E} \cdot \overline{da}}_{\int_V \rho \, dV}$$

$$\underbrace{\oint_S \bar{J} \cdot \overline{da}}_{\substack{\text{Total current}\\ \text{leaving volume}\\ \text{through surface}}} + \frac{d}{dt} \underbrace{\int_V \rho \, dV}_{\substack{\text{Total charge}\\ \text{inside volume}}} = 0$$

6. Lorentz Force Law

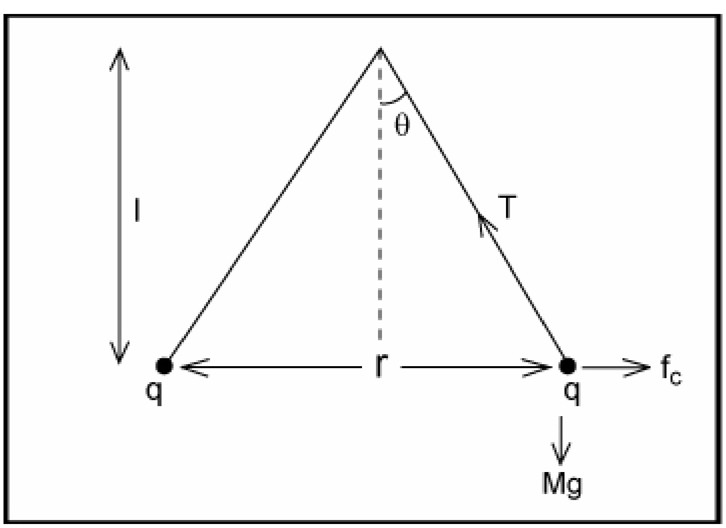
$$\bar{f} = q\left(\bar{E} + \bar{v} \times \mu_0 \bar{H}\right)$$

II. <u>Electric Field from Point Charge</u>



$$\oint_S \varepsilon_0 \bar{E} \cdot \overline{da} = \varepsilon_0 E_r 4\pi r^2 = q$$

$$E_r = \frac{q}{4\pi \varepsilon_0 r^2}$$



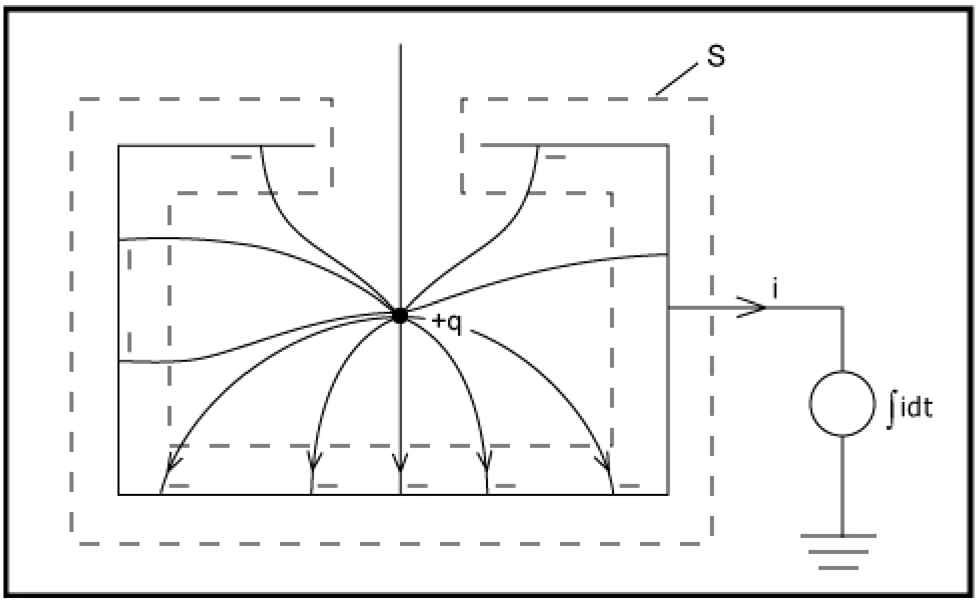
$$T \sin\theta = f_c = \frac{q^2}{4\pi \varepsilon_0 r^2}$$

$$T \cos\theta = Mg$$

$$\tan\theta = \frac{q^2}{4\pi \varepsilon_0 r^2 Mg} = \frac{r}{2l}$$

$$q = \left[\frac{2\pi\varepsilon_0 r^3 Mg}{l}\right]^{\frac{1}{2}}$$

III. Faraday Cage



$$\oint_S \bar{J} \cdot \overline{da} = i = -\frac{d}{dt}\int \rho\, dV = -\frac{d}{dt}(-q) = \frac{dq}{dt}$$

$$\int i\,dt = q$$

IV. Edgerton's Boomer

1. Magnetic Field, Current, and Inductance

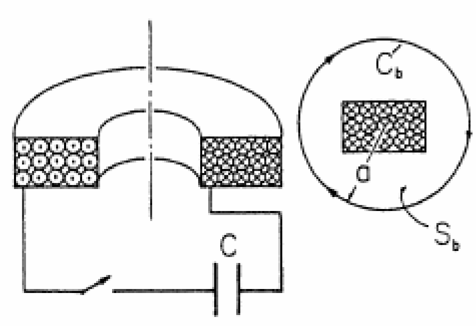


Figure 10.2.2 When the spark gap switch is closed, the capacitor discharges into the coil. The contour C_b is used to estimate the average magnetic field intensity that results.

Courtesy of Hermann A. Haus and James R. Melcher. Used with permission.

$$\oint_{C_b} \mathbf{H} \cdot d\mathbf{s} \approx H_1 2\pi a = N_1 i_1 \Rightarrow H_1 \approx \frac{N_1 i_1}{2\pi a}$$

$$\lambda \approx N_1 \left(\pi a^2\right) \mu_0 H_1 = \frac{N_1^2 \cancel{\pi} a^{\cancel{2}} \mu_0}{2\pi \cancel{a}} i_1 \approx \frac{N_1^2 a \mu_0}{2} i_1$$

$$L = \frac{\lambda}{i_1} \approx \frac{N_1^2 a \mu_0}{2}$$

$$\omega = \frac{1}{\sqrt{LC}}$$

$$\frac{1}{2} L i_p^2 \approx \frac{1}{2} C v_p^2 \Rightarrow i_p \approx v_p \sqrt{C/L}$$

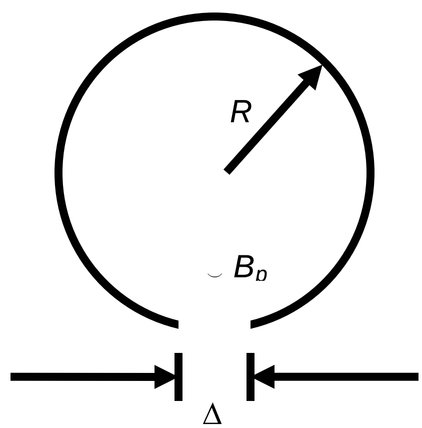
$C = 25\ \mu f,\ v_p = 4\ kV,\ N_1 = 50,\ a \approx 7\ cm$

$$L_1 \approx 0.1\ mH$$

$$i_p \approx 2000\,A,\ \omega \approx 20 \times 10^3 / s \Rightarrow f = \frac{\omega}{2\pi} \approx 3\,kHz$$

$$H_p \approx 2.3 \times 10^5\ A/m \Rightarrow B_p = \mu_0 H_p \approx 0.3\ \text{Teslas} \approx 3000\ \text{Gauss}$$

2. Electrical Breakdown in Single Turn Coil with Small Gap



$$E \approx \begin{cases} 0 & \text{Inside Metal Coil} \\ E_0 & \text{Small Gap } \Delta \end{cases}$$

$$\oint_C \bar{E}\cdot d\bar{s} = E_0\Delta = -\frac{d}{dt}(B_p \pi R^2)$$

$$B_p = B_m \cos\omega t$$

$$E_0 = \frac{B_m \omega \pi R^2}{\Delta}\sin\omega t$$

Take: $B_m \approx 0.3$ Tesla, $\omega \approx 20{,}000$ radians/second, $R \approx 0.07$ m, $\Delta = 0.01$ mm

$$E_m = \frac{B_m \omega \pi R^2}{\Delta} = \frac{0.3(20{,}000)\pi(0.07)^2}{10^{-5}} = 9\times 10^6 \text{ Volts/meter}$$

Breakdown strength of air $\approx 3\times 10^6$ Volts/meter.

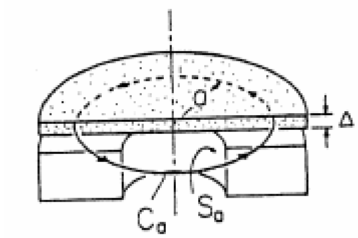


Figure 10.2.3 Metal disk placed on top of coil shown in Figure 10.2.2.

3. Force on Metal Disk

$$\oint_{C_b} \bar{E}\cdot\overline{ds} \approx 2\pi a E_\phi = -\frac{d}{dt}\int_{S_a} \bar{B}\cdot\overline{da} \approx -\pi a^2 \frac{dB_p}{dt} = \pi a^2 B_m \omega \sin\omega t$$

$$J_\phi = \sigma E_\phi = -\frac{\sigma a}{2}\frac{dB_p}{dt} = \frac{\sigma a}{2} B_m \omega \sin\omega t$$

$$\bar{F} = \bar{J} \times \mu_0 \bar{H}\ , \qquad \bar{f} = \int_V \bar{F}\, dV = \int_V \bar{J} \times \mu_0 \bar{H}\, dV$$

Force per unit volume ($\bar{F}$) — total force ($\bar{f}$)

$$K_\phi \approx J_\phi \Delta = -H_r \Rightarrow H_r = -J_\phi \Delta$$

$$\bar{F} = \bar{J}\times \mu_0 \bar{H} = J_\phi \bar{i}_\phi \times \mu_0 H_r \bar{i}_r = -\mu_0 J_\phi H_r \bar{i}_z = \mu_0 J_\phi^2 \Delta \bar{i}_z$$

$$F_z = \mu_0 J_\phi^2 \Delta = \mu_0 \Delta\left(\frac{\sigma a}{2} B_m \omega\right)^2 \sin^2\omega t$$

$$f_z = F_z \pi a^2 \Delta = \pi \frac{\mu_0 \Delta^2 \sigma^2 a^4}{4} B_m^2 \omega^2 \sin^2\omega t$$

$\sigma_{aluminum} \approx 3.7 \times 10^7$ Siemens/meter, a=0.07 m, Δ=2 mm, $\omega = 20{,}000$ radians/second, $B_m \approx 0.3$ Tesla, M=0.08 kg

$$f_z = \frac{\mu_0}{4\pi}\left(\pi\Delta\sigma a^2\omega B_m\right)^2 \sin^2 \omega t$$
$$= 10^{-7}\left[\pi\left(2\times 10^{-3}\right)\left(3.7\times 10^{7}\right)\left(.07\right)^2 20{,}000\left(0.3\right)\right]^2 \sin^2 \omega t$$
$$= 4.7\times 10^6 \sin^2 \omega t$$

$$Mg = (0.08)9.8 \approx 0.8 \text{ Newtons}$$
$$\frac{f_{max}}{Mg} \approx \frac{4.7\times 10^6}{0.8} \approx 5.9\times 10^6$$

Neglecting losses:

$$\frac{1}{2}CV^2 = \frac{1}{2}Mv^2(t=0_+) = Mgh$$
$$v(t=0_+) = \sqrt{\frac{C}{M}}V$$
$$C = 25\mu f\text{ , } M = .08 \text{ kg, } V_p = 4000 \text{ volts}$$
$$v(t=0_+) = 70.7 \text{ meters/second} \qquad \text{(Initial velocity)}$$
$$h = \frac{v^2}{2g}\left(t=0_+\right) = 255 \text{ meters} \qquad \text{(Maximum height)}$$

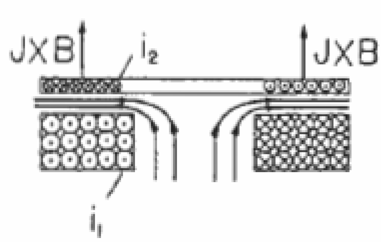


Figure 10.2.4 Currents induced in the metal disk tend to induce a field that bucks out that imposed by the driving coil. These currents result in a force on the disk that tends to propel it upward.

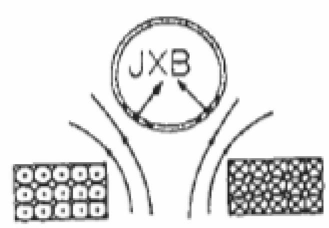


Figure 10.4.4 In an experiment giving evidence of the currents induced when a field is suddenly applied transverse to a conducting cylinder, an aluminum foil cylinder, subjected to the field produced by the experiment of Figure 10.2.2, is crushed.