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6.013/ESD.013J Electromagnetics and Applications, Fall 2005

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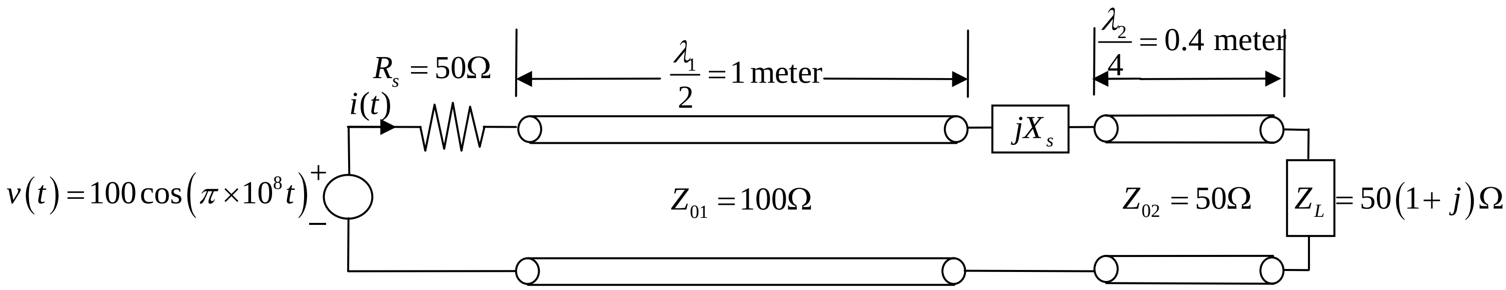
Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
6.013 Electromagnetics and Applications
Quiz 2, November 17, 2005

6.013 Formula Sheets attached.

Problem 1



A transmission line system incorporates two transmission lines with characteristic impedances of $Z_{01} = 100\Omega$ and $Z_{02} = 50\Omega$ as illustrated above. A voltage source is applied at the left end, $v(t) = 100\cos(\pi \times 10^8 t)$. At this frequency, line 1 has length of $\frac{\lambda_1}{2} = 1\,\text{meter}$ and line 2 has length of $\frac{\lambda_2}{4} = 0.4\,\text{meter}$, where λ_1 and λ_2 are the wavelengths along each respective transmission line. The two transmission lines are connected by a series reactance jX_s and the end of line 2 is loaded by impedance $Z_L = 50(1+j)\Omega$. The voltage source is connected to line 1 through a source resistance $R_s = 50\Omega$.

a) What are the speeds c_1 and c_2 of electromagnetic waves on each line?

b) It is desired that X_s be chosen so that the source current $i(t) = I_0\cos(\pi \times 10^8 t)$ is in phase with the voltage source. What is X_s?

c) For the value of X_s in part (b), what is the peak amplitude I_0 of the source current $i(t)$? Note that the value of X_s itself is not needed to answer this question or part (d).

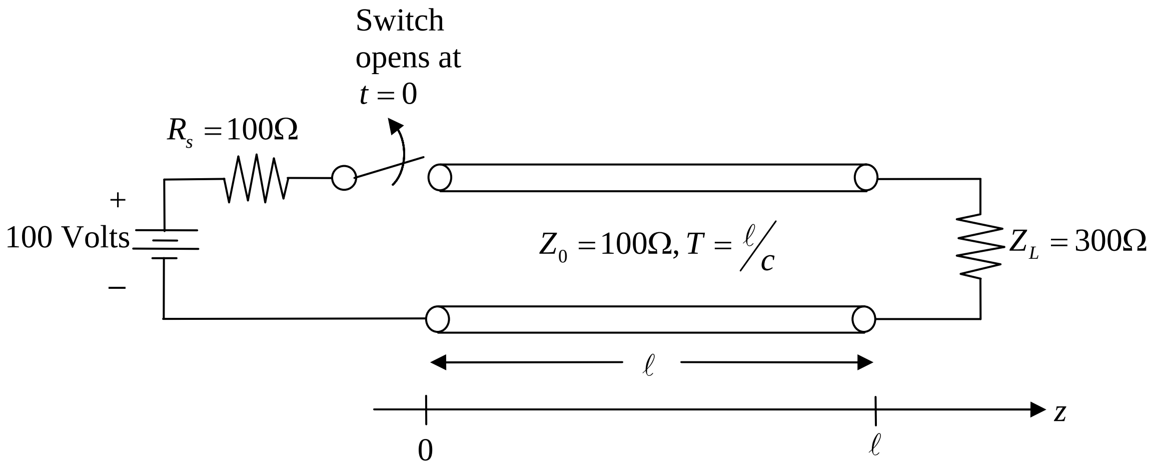
Problem 2

A parallel plate waveguide is to be designed so that only TEM modes can propagate in the frequency range $0 < f < 2\,\text{GHz}$. The dielectric between the plates has a relative dielectric constant of $\varepsilon_r = 9$ and a magnetic permeability of free space μ_0.

a) What is the maximum allowed spacing $d_{\max}$ between the parallel plate waveguide plates?

b) If the plate spacing is 2.1 cm, and $f =$ 10 GHz, what TE_n and TM_n modes will propagate?

Problem 3



A transmission line of length ℓ, characteristic impedance $Z_0 = 100\Omega$, and one-way time of flight $T = \ell / c$ is connected at $z = 0$ to a 100 volt DC battery through a series source resistance $R_s = 100\Omega$ and a switch . The $z = \ell$ end is loaded by a 300Ω resistor.

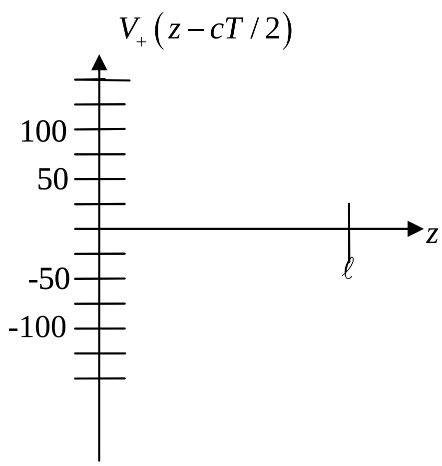
a) The switch at the $z = 0$ end has been closed for a very long time so that the system is in the DC steady state. What are the values of the positive and negative traveling wave voltage amplitudes $V_+(z - ct)$ and $V_-(z + ct)$?

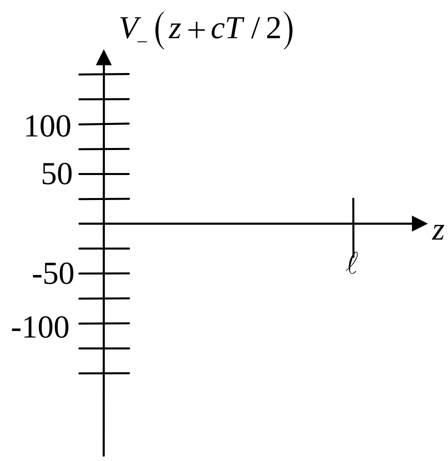
Part b, on the next page, to be handed in with your exam. Put your name at the top of the next page.

Name: ______________________

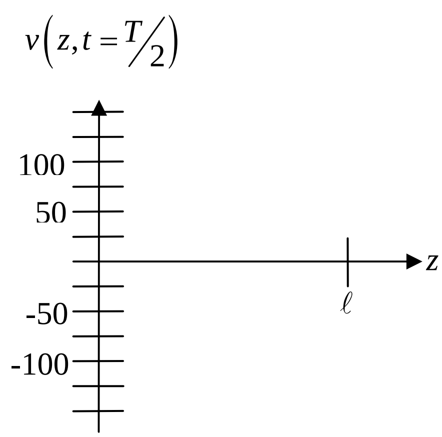
b) With the system in the DC steady state, the switch is suddenly opened at time $t = 0$.

i) Plot the positive and negative traveling wave voltage amplitudes, $V_+(z - ct)$ and $V_-(z + ct)$, as a function of z at time $t = T/2$.





ii) Plot the transmission line voltage $v(z,t)$ as a function of z at time $t = T/2$.



Please tear out this page and hand in with your exam. Don't forget to put your name at the top of this page.

6.013 Quiz 2 Formula Sheet
November 17, 2005

Cartesian Coordinates (x,y,z):

$$\nabla\Psi = \hat{x}\frac{\partial\Psi}{\partial x} + \hat{y}\frac{\partial\Psi}{\partial y} + \hat{z}\frac{\partial\Psi}{\partial z}$$

$$\nabla\bullet\overline{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla\times\overline{A} = \hat{x}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{y}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{z}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\nabla^2\Psi = \frac{\partial^2\Psi}{\partial x^2} + \frac{\partial^2\Psi}{\partial y^2} + \frac{\partial^2\Psi}{\partial z^2}$$

Cylindrical coordinates (r,ϕ,z):

$$\nabla\Psi = \hat{r}\frac{\partial\Psi}{\partial r} + \hat{\phi}\frac{1}{r}\frac{\partial\Psi}{\partial\phi} + \hat{z}\frac{\partial\Psi}{\partial z}$$

$$\nabla\bullet\overline{A} = \frac{1}{r}\frac{\partial(rA_r)}{\partial r} + \frac{1}{r}\frac{\partial A_\phi}{\partial\phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla\times\overline{A} = \hat{r}\left(\frac{1}{r}\frac{\partial A_z}{\partial\phi} - \frac{\partial A_\phi}{\partial z}\right) + \hat{\phi}\left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right) + \hat{z}\frac{1}{r}\left(\frac{\partial(rA_\phi)}{\partial r} - \frac{\partial A_r}{\partial\phi}\right) = \frac{1}{r}\det\begin{vmatrix} \hat{r} & r\hat{\phi} & \hat{z} \\ \partial/\partial r & \partial/\partial\phi & \partial/\partial z \\ A_r & rA_\phi & A_z \end{vmatrix}$$

$$\nabla^2\Psi = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\Psi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\Psi}{\partial\phi^2} + \frac{\partial^2\Psi}{\partial z^2}$$

Spherical coordinates (r,θ,ϕ):

$$\nabla\Psi = \hat{r}\frac{\partial\Psi}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial\Psi}{\partial\theta} + \hat{\phi}\frac{1}{r\sin\theta}\frac{\partial\Psi}{\partial\phi}$$

$$\nabla\bullet\overline{A} = \frac{1}{r^2}\frac{\partial(r^2A_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta A_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$$

$$\nabla\times\overline{A} = \hat{r}\frac{1}{r\sin\theta}\left(\frac{\partial(\sin\theta A_\phi)}{\partial\theta} - \frac{\partial A_\theta}{\partial\phi}\right) + \hat{\theta}\left(\frac{1}{r\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{1}{r}\frac{\partial(rA_\phi)}{\partial r}\right) + \hat{\phi}\frac{1}{r}\left(\frac{\partial(rA_\theta)}{\partial r} - \frac{\partial A_r}{\partial\theta}\right)$$

$$= \frac{1}{r^2\sin\theta}\det\begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\,\hat{\phi} \\ \partial/\partial r & \partial/\partial\theta & \partial/\partial\phi \\ A_r & rA_\theta & r\sin\theta A_\phi \end{vmatrix}$$

$$\nabla^2\Psi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Psi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\Psi}{\partial\phi^2}$$

Gauss' Divergence Theorem:

$$\int_V \nabla\bullet\overline{G}\,dv = \oint_A \overline{G}\bullet\hat{n}\,da$$

Stokes' Theorem:

$$\int_A (\nabla\times\overline{G})\bullet\hat{n}\,da = \oint_C \overline{G}\bullet d\overline{\ell}$$

Vector Algebra:

$$\nabla = \hat{x}\partial/\partial x + \hat{y}\partial/\partial y + \hat{z}\partial/\partial z$$

$$\overline{A}\bullet\overline{B} = A_xB_x + A_yB_y + A_zB_z$$

$$\nabla\bullet(\nabla\times\overline{A}) = 0$$

$$\nabla\times(\nabla\times\overline{A}) = \nabla(\nabla\bullet\overline{A}) - \nabla^2\overline{A}$$

Basic Equations for Electromagnetics and Applications

Fundamentals

$\overline{f} = q\left(\overline{E} + \overline{v} \times \mu_o \overline{H}\right)[N]$

$\nabla \times \overline{E} = -\partial \overline{B}/\partial t$

$\oint_c \overline{E} \bullet d\overline{s} = -\frac{d}{dt}\int_A \overline{B} \bullet d\overline{a}$

$\nabla \times \overline{H} = \overline{J} + \partial \overline{D}/\partial t$

$\oint_c \overline{H} \bullet d\overline{s} = \int_A \overline{J} \bullet d\overline{a} + \frac{d}{dt}\int_A \overline{D} \bullet d\overline{a}$

$\nabla \bullet \overline{D} = \rho \rightarrow \oint_A \overline{D} \bullet d\overline{a} = \int_V \rho dv$

$\nabla \bullet \overline{B} = 0 \rightarrow \oint_A \overline{B} \bullet d\overline{a} = 0$

$\nabla \bullet \overline{J} = -\partial \rho/\partial t$

$\overline{E}$ = electric field (Vm^{-1})

$\overline{H}$ = magnetic field (Am^{-1})

$\overline{D}$ = electric displacement (Cm^{-2})

$\overline{B}$ = magnetic flux density (T)

Tesla (T) = Weber m^{-2} = 10,000 gauss

ρ = charge density (Cm^{-3})

$\overline{J}$ = current density (Am^{-2})

σ = conductivity (Siemens m^{-1})

$\overline{J}_s$ = surface current density (Am^{-1})

ρ_s = surface charge density (Cm^{-2})

$\varepsilon_o \approx 8.854 \times 10^{-12}\ Fm^{-1}$

$\mu_o = 4\pi \times 10^{-7}\ Hm^{-1}$

$c = (\varepsilon_o\mu_o)^{-0.5} \cong 3 \times 10^8\ ms^{-1}$

$e = -1.60 \times 10^{-19}$ C

$\eta_o \cong 377$ ohms $= (\mu_o/\varepsilon_o)^{0.5}$

$\left(\nabla^2 - \mu\varepsilon\partial^2/\partial t^2\right)\overline{E} = 0$ [Wave Eqn.]

$E_y(z,t) = E_+(z-ct) + E_-(z+ct) = \mathrm{Re}\{\underline{E}_y(z)e^{j\omega t}\}$

$H_x(z,t) = \eta_o^{-1}[E_+(z-ct)-E_-(z+ct)]$ [or(ωt-kz) or (t-z/c)]

$\oint_A \left(\overline{E} \times \overline{H}\right) \bullet d\overline{a} + (d/dt)\int_V \left(\varepsilon|\overline{E}|^2/2 + \mu|\overline{H}|^2/2\right) dv$

$= -\int_V \overline{E} \bullet \overline{J}\ dv$ (Poynting Theorem)

Media and Boundaries

$\overline{D} = \varepsilon_o \overline{E} + \overline{P}$

$\nabla \bullet \overline{D} = \rho_f,\ \tau = \varepsilon/\sigma$

$\nabla \bullet \varepsilon_o \overline{E} = \rho_f + \rho_p$

$\nabla \bullet \overline{P} = -\rho_p,\ \overline{J} = \sigma \overline{E}$

$\overline{B} = \mu \overline{H} = \mu_o\left(\overline{H} + \overline{M}\right)$

$\varepsilon = \varepsilon_o\left(1 - \omega_p^{\ 2}/\omega^2\right),\quad \omega_p = \left(Ne^2/m\varepsilon_o\right)^{0.5}$ **(Plasma)**

$\varepsilon_{eff} = \varepsilon(1 - j\sigma/\omega\varepsilon)$

skin depth $\delta = (2/\omega\mu\sigma)^{0.5}$ [m]

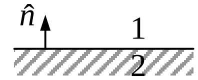
$\overline{E}_{1//} - \overline{E}_{2//} = 0$

$\overline{H}_{1//} - \overline{H}_{2//} = \overline{J}_s \times \hat{n}$

$B_{1\perp} - B_{2\perp} = 0$

$D_{1\perp} - D_{2\perp} = \rho_s$

↳ 0 = if $\sigma = \infty$



Electromagnetic Waves

$\left(\nabla^2 - \mu\varepsilon\partial^2/\partial t^2\right)\overline{E} = 0$ [Wave Eqn.]

$\left(\nabla^2 + k^2\right)\underline{E} = 0,\ \underline{E} = \underline{E}_o e^{-j\overline{k}\bullet\overline{r}}$

$k = \omega(\mu\varepsilon)^{0.5} = \omega/c = 2\pi/\lambda$

$k_x^{\ 2} + k_y^{\ 2} + k_z^{\ 2} = k_o^{\ 2} = \omega^2\mu\varepsilon$

$v_p = \omega/k,\ v_g = (\partial k/\partial\omega)^{-1}$

$\theta_r = \theta_i$

$\sin\theta_t/\sin\theta_i = k_i/k_t = n_i/n_t$

$\theta_c = \sin^{-1}\left(n_t/n_i\right)$

$\theta_B = \tan^{-1}\left(\varepsilon_t/\varepsilon_i\right)^{0.5}$ for TM

$\theta > \theta_c \Rightarrow \underline{E}_t = \underline{E}_i \underline{T} e^{+\alpha x - jk_z z}$

$\underline{k} = k' - jk''$

$\underline{\Gamma} = \underline{T} - 1$

$\underline{T}_{TE} = 2/\left(1 + \left[\eta_i \cos\theta_t/\eta_t \cos\theta_i\right]\right)$

$\underline{T}_{TM} = 2/\left(1 + \left[\eta_t \cos\theta_t/\eta_i \cos\theta_i\right]\right)$

Transmission Lines

Time Domain

$\partial v(z,t)/\partial z = -L\partial i(z,t)/\partial t$

$\partial i(z,t)/\partial z = -C\partial v(z,t)/\partial t$

$\partial^2 v/\partial z^2 = LC\ \partial^2 v/\partial t^2$

$v(z,t) = V_+(t - z/c) + V_-(t + z/c)$

$i(z,t) = Y_o[V_+(t - z/c) - V_-(t + z/c)]$

$c = (LC)^{-0.5} = (\mu\varepsilon)^{-0.5}$

$Z_o = Y_o^{-1} = (L/C)^{0.5}$

$\Gamma_L = V_-/V_+ = (R_L - Z_o)/(R_L + Z_o)$

Frequency Domain

$(d^2/dz^2 + \omega^2 LC)\underline{V}(z) = 0$

$\underline{V}(z) = \underline{V}_+e^{-jkz} + \underline{V}_-e^{+jkz}$

$\underline{I}(z) = Y_o[\underline{V}_+e^{-jkz} - \underline{V}_-e^{+jkz}]$

$k = 2\pi/\lambda = \omega/c = \omega(\mu\varepsilon)^{0.5}$

$\underline{Z}(z) = \underline{V}(z)/\underline{I}(z) = Z_o\underline{Z}_n(z)$

$\underline{Z}_n(z) = \left[1 + \underline{\Gamma}(z)\right]/\left[1 - \underline{\Gamma}(z)\right] = R_n + jX_n$

$\underline{\Gamma}(z) = \left(\underline{V}_-/\underline{V}_+\right)e^{2jkz} = \left[\underline{Z}_n(z) - 1\right]/\left[\underline{Z}_n(z) + 1\right]$

$\underline{Z}(z) = Z_o\left(\underline{Z}_L - jZ_o \tan kz\right)/\left(\underline{Z}_o - jZ_L \tan kz\right)$

$\mathrm{VSWR} = |\underline{V}_{max}|/|\underline{V}_{min}|$