Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
6.013 Electromagnetics and Applications

Problem Set #5 | Issued: 10/4/05
Fall Term 2005 | Due: 10/12/05

Reading Assignment: Sections 1.3.2, 1.4, 1.6, 5.1, 5.3
Quiz 1 on Thursday, October 20 at 10-11 a.m. Will cover material through Problem Set #5.
The **Final Exam** will be on Dec. 21, 1:30-4:30 p.m.

Problem 5.1

For the following electric fields in a linear medium of constant dielectric permittivity ε and magnetic permeability μ, find the free charge density ρ_f, magnetic field $\overline{H}$, and current density $\overline{J}$.

a) $\overline{E} = E_0 (x\overline{i}_x + y\overline{i}_y) \sin \omega t$

b) $\overline{E} = E_0 (y\overline{i}_x - x\overline{i}_y) \cos \omega t$

Problem 5.2

An electric field is of the form:

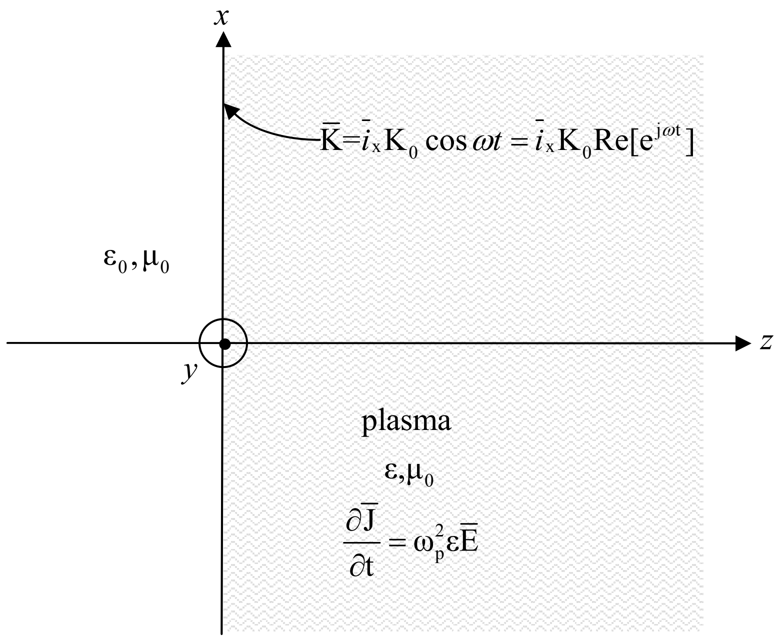
$$\overline{E} = 10 \mathrm{Re}\left[e^{j(4\pi \times 10^6 t - 4\pi \times 10^{-2} z)}\right]\overline{i}_x \qquad \text{volts/meter}$$

a) What is the frequency f, wavelength λ, and speed of light in the medium?

b) If the medium has magnetic permeability $\mu_0 = 4\pi \times 10^{-7}$ henry/meter, what are the relative permittivity ε_r, wave impedance η, and the magnetic field $\overline{H}$?

c) What is the Poynting vector, $\overline{S} = \overline{E} \times \overline{H}$?

Problem 5.3



A sheet of surface current with the surface current density:

$$\overline{K}=\bar{i}_x K_0 \cos\omega t = \bar{i}_x K_0 \mathrm{Re}[e^{j\omega t}] \qquad \text{amperes/meter}$$

is located at $z = 0$ with free space extending from $-\infty < z < 0$ and a plasma extending for $z > 0$. The plasma region has dielectric permittivity ε and magnetic permeability μ_0 of free space with current constitutive law:

$$\frac{\partial \overline{J}}{\partial t} = \omega_p^2 \varepsilon \overline{E}$$

a) If all fields are of the form:

$$\overline{J}(z,t)=\mathrm{Re}[\hat{\overline{J}}(z)e^{j\omega t}]$$
$$\overline{E}(z,t)=\mathrm{Re}[\hat{\overline{E}}(z)e^{j\omega t}]$$
$$\overline{H}(z,t)=\mathrm{Re}[\hat{\overline{H}}(z)e^{j\omega t}]$$

what is the plasma complex conductivity, $\sigma(\omega)$, in the plasma region defined as:

$$\hat{\overline{J}}(z)=\sigma(\omega)\hat{\overline{E}}(z) \ ?$$

b) Ampere's law in the plasma region becomes:

$$\nabla \times \hat{\bar{H}} = \hat{\bar{J}}(z) + j\omega\varepsilon\hat{\bar{E}} = j\omega\varepsilon(\omega)\hat{\bar{E}} \ .$$

What is the frequency dependent permittivity $\varepsilon(\omega)$?

c) Assume that $\hat{\bar{E}}(z)$ is of the form:

$$\hat{\bar{E}}(z)=\begin{cases}\hat{E}_p e^{-jk_p z}\bar{i}_x & z>0\\ \hat{E}_0 e^{jk_0 z}\bar{i}_x & z<0\end{cases}$$

What are the wavenumbers k_p and k_0?

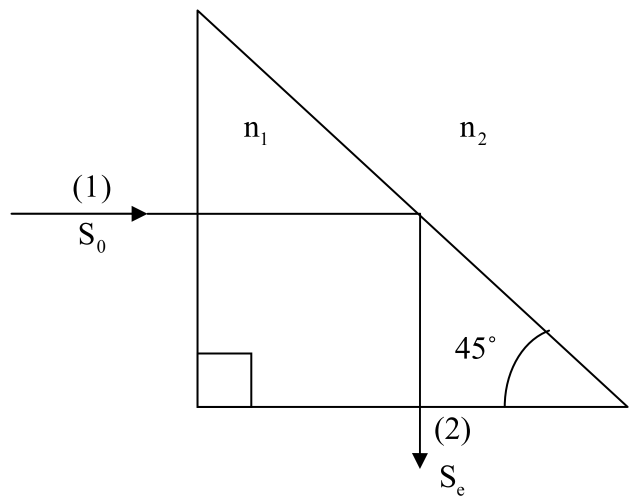
d) What is the general form of $\hat{\bar{H}}(z)$ in each region in terms of $\hat{E}_0$ and $\hat{E}_p$?

e) What are the boundary conditions at $z=0$ on the electric and magnetic fields?

f) Solve for $\hat{E}_0$, $\hat{E}_p$, $\hat{\bar{E}}(z)$, and $\hat{\bar{H}}(z)$.

g) Find the time average Poynting vector, $<\bar{S}>=\frac{1}{2}\mathrm{Re}\left[\hat{\bar{E}}\times\hat{\bar{H}}^*\right]$, for $z<0$ and $z>0$ and evaluate for $\omega>\omega_p$ and $\omega<\omega_p$.

Problem 5.4



A glass prism in the shape of an isosceles right triangle has an index of refraction n_1. The surrounding environment has index of refraction n_2. Neglect multiple internal reflections within the prism.

a) What is the minimum value of n_1 for no time average power to be transmitted across the prism hypotenuse when the prism is in free space $(n_2 = 1)$ or in water $(n_2 = 1.33)$?

b) If the incident light at point (1) has time average power per unit area S_0, what is the exiting time average power S_e at point (2) in terms of the refractive index ratio $n=n_1/n_2$ assuming that n_1 is above the minimum value for no time average power to be transmitted across the prism hypotenuse. Evaluate S_0/S_e for the two minimum values of n_1 found in part (a) for free space or water surrounding the prism.