Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
6.641 Electromagnetic Fields, Forces, and Motion

Problem Set #1	Issued: 2/1/05
Spring Term 2005	Due: 2/10/05

Suggested Reading: Zahn – Section 1.4, 1.5, 2.1-2.4, 3.3, 5.1, 5.2, 5.6

Problem 1.1

In Demonstration 1.4.1 the magnetic field strength is measured by a Hall effect probe. This probe works by the principle that when charges flow perpendicular to a magnetic field, the transverse displacement due to the Lorentz force can give rise to an electric field.

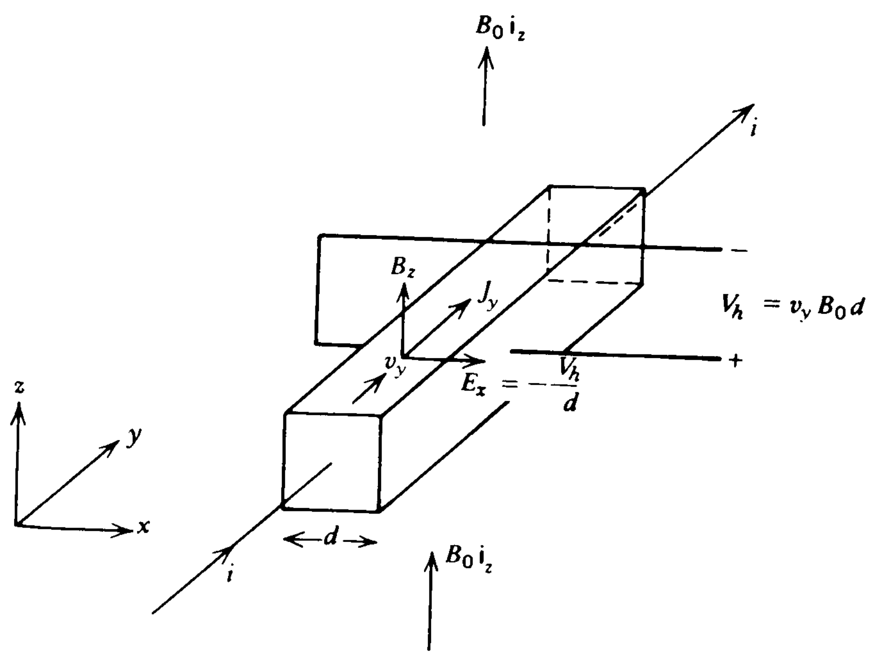


Figure 5-6 A magnetic field perpendicular to a current flow deflects the charges transversely giving rise to an electric field and the Hall voltage. The polarity of the voltage is the same as the sign of the charge carriers.

(a) A uniform magnetic field $\overline{B} = B_o \bar{i}_z = \mu_o H_o \bar{i}_z$ is applied to a material carrying a current in the *y* direction. For positive charges, as for holes in a p-type semiconductor, the charge velocity $\overline{v} = v_y \bar{i}_y$ is in the positive y direction, while for negative charges as typically occur in metals or in n-type semiconductors, the charge velocity v_y is negative. In the steady state, the charge velocity v_y does not vary with time so the net force on the charges must be zero. What is the electric field (magnitude and direction) in terms of v_y and B_o?

(b) What is the Hall voltage, $V_H = \Phi(x = d) - \Phi(x = 0)$ in terms of v_y, B_o and *d*?

(c) Can this measurement determine the polarity of the charge carriers assuming that the current *i* is positively *y*-directed and B_0 is positively *z*-directed?

Problem 1.2

In a spherically symmetric configuration, the region $r < b$ has the uniform charge density ρ_b and is surrounded by a region $b < r < a$ having the uniform charge density ρ_a. At $r = b$ there is no surface charge density, while at $r = a$ there is perfectly conducting sheet with surface charge density that assures $\boldsymbol{E} = 0$ for $r > a$.

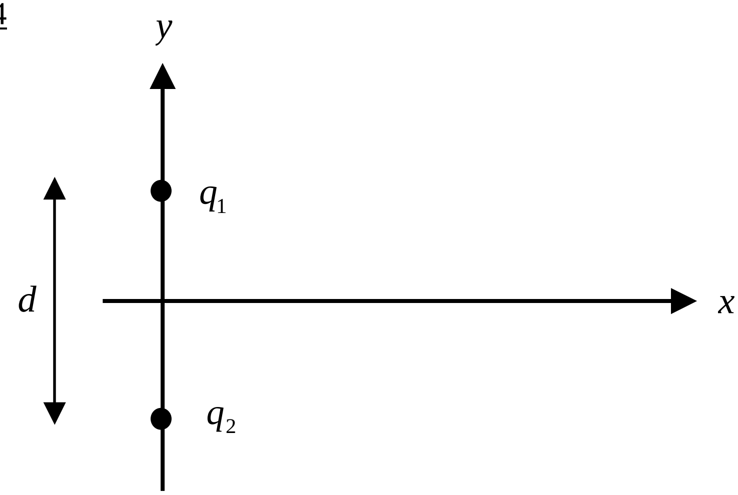
(a) Determine $\boldsymbol{E}$ in the two regions $r < b$ and $b < r < a$.
(b) What is the surface charge density at $r = a$?
(c) What is the total charge in the regions $r < b$ and $b < r < a$? Including the surface charge at $r = a$, what is the total charge in the system?

Problem 1.3

In polar coordinates, a uniform current density $J_o \bar{\boldsymbol{i}}_{\boldsymbol{z}}$ exists over the cross-section of a wire having a radius b. The total current in the wire is returned in the -z direction as a uniform surface current at the radius $r = a > b$.

(a) What is the surface current density at $r = a$?
(b) Find the magnetic field in the regions $0 < r < b$, $b < r < a$, and $r > a$.
(c) Is the boundary condition on $\bar{H}$ consistent with part (a)?

Problem 1.4



(a) Two point charges are a distance d apart along the y-axis. Find the electric field (magnitude and direction) at any point in the $y=0$ plane when the charges are:
   i) $q_1 = q, \quad q_2 = 0$
   ii) both equal $q_1 = q_2 = q$
   iii) of opposite polarity but equal magnitude $q_1 = -q_2 = q$.

   This configuration is called an electric dipole with dipole moment $p_y = qd$.
(b) For each of the 3 cases in (a) find the force on q_1.