6.642 Continuum Electromechanics
Fall 2008

Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
6.642 Continuum Electromechanics

Problem Set #3 — Issued: 9/17/08
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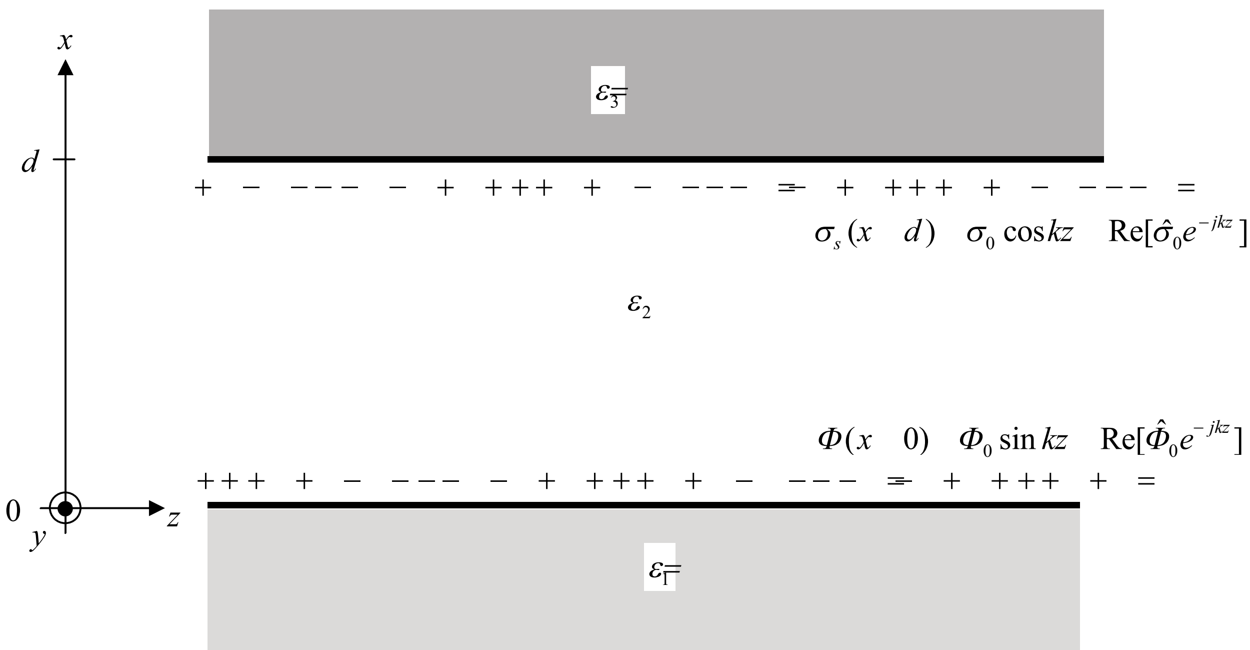
Problem 1

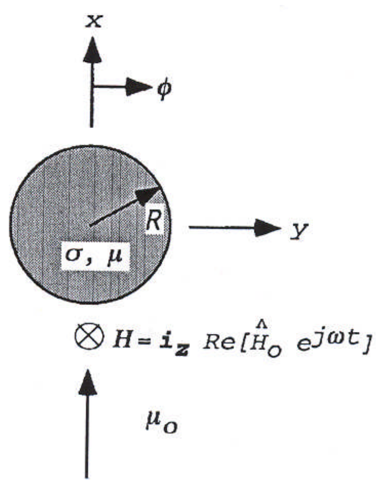


A potential sheet, $\Phi(x = 0) = \Phi_0 \sin kz = \mathrm{Re}[\hat{\Phi}_0 e^{-jkz}]$, is located at $x = 0$ between two dielectric media with dielectric permittivity ε_1 for $-\infty < x < 0$ and ε_2 for $0 < x < d$. A sheet of surface charge, $\sigma_s(x = d) = \sigma_0 \cos kz = \mathrm{Re}[\hat{\sigma}_0 e^{-jkz}]$, is located at $x = d$ between two dielectric media with dielectric permittivity ε_2 for $0 < x < d$ and ε_3 for $d < x < \infty$. All materials are lossless. The system is of infinite extent in the y direction.

a) Find the complex amplitudes $\hat{\sigma}_0$ and $\hat{\Phi}_0$ in terms of σ_0 and Φ_0.
b) What is the complex amplitude of the potential $\hat{\Phi}(x = d)$ along the sheet of surface charge at $x = d$?
c) What is the complex amplitude of the surface charge density along the potential sheet at $x = 0$? (Express your answer in terms of $\hat{\sigma}_0$, $\hat{\Phi}_0$, and $\hat{\Phi}(x = d)$).
d) What is the space average force per unit area in the x and z directions on the sheet of surface charge at $x = d$? Use the results of part (a) to greatly simplify your answer. Hint: The Maxwell Stress tensor will be the easiest way to solve for the space average free charge and polarization forces in the x and z directions.

Problem 2

An infinitely long cylinder of radius R, conductivity σ, and magnetic permeability μ is placed in a uniform magnetic field $\bar{H} = \bar{i}_z \mathrm{Re}[\hat{H}_0 e^{j\omega t}]$. The region $r > R$ is free space.



The governing equation is the magnetic diffusion equation.

$$\nabla^2 \bar{H} = \mu\sigma \frac{\partial \bar{H}}{\partial t}$$

There is no surface current on the $r = R$ interface.

a) Assume that

$$\bar{H}(r,t) = \mathrm{Re}[\hat{H}_z(r) e^{j\omega t}]\bar{i}_z$$

and show that the governing equation is

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\hat{H}_z}{dr}\right) = j\omega\mu\sigma \hat{H}_z$$

b) Solve part (a) for $\hat{H}_z(r)$ for $r < R$. What boundary conditions must the solution satisfy?

Hint 1: The solution to Bessel's Equation

$$r\frac{d}{dr}\left(r\frac{d\hat{H}_z}{dr}\right) + (k^2 r^2 - n^2)\hat{H}_z = 0$$

is

$$\hat{H}_z(r) = C_1 J_n(kr) + C_2 Y_n(kr)$$

where J_n is called a Bessel function of the first kind of order n and Y_n is called the n^{th} order Bessel function of the second kind. Note that $J_n(0)$ is finite while $Y_n(0)$ is infinite. Note also that k has two solutions. Which solution for k can be used to solve for $\hat{H}_z(r)$?

Hint 2: $J_0(x) = 1 - \frac{x^2}{2^2} + \frac{x^4}{2^4(2!)^2} - \frac{x^6}{2^6(3!)^2} + \ldots$

c) What is the current density $\bar{J}(r)$?

Hint: $z\frac{d}{dz}J_n(z) = nJ_n(z) - zJ_{n+1}(z)$

d) Plot $H_z(r, t = 0)$ and $\bar{J}(r, t = 0)$ for $\delta/R = 0.05,\ 0.1,\ 0.25,\ 0.5,\ 0.75,\ 1,\ \infty$ where $\delta = \sqrt{2/(\omega\mu\sigma)}$ is the skin-depth.