

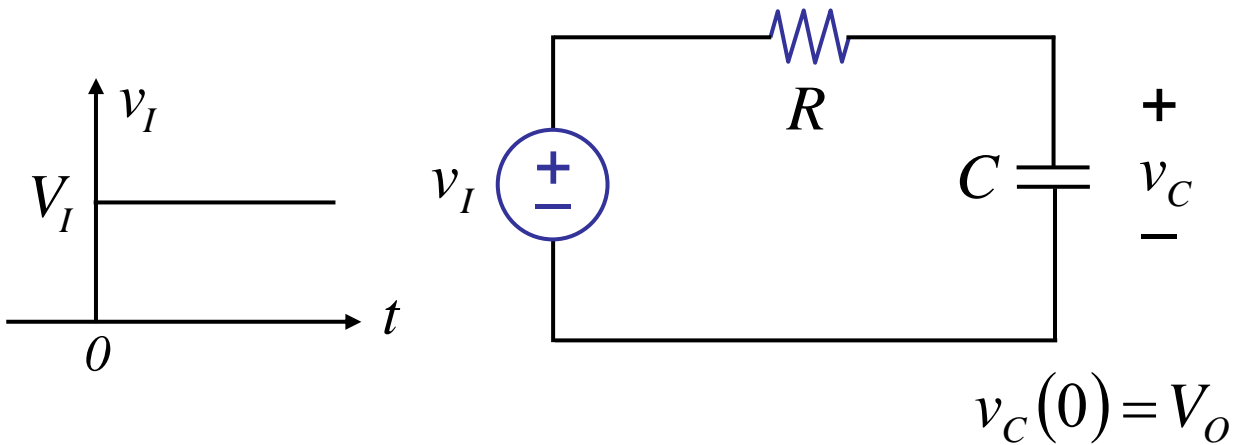
**6.002**

**CIRCUITS AND  
ELECTRONICS**

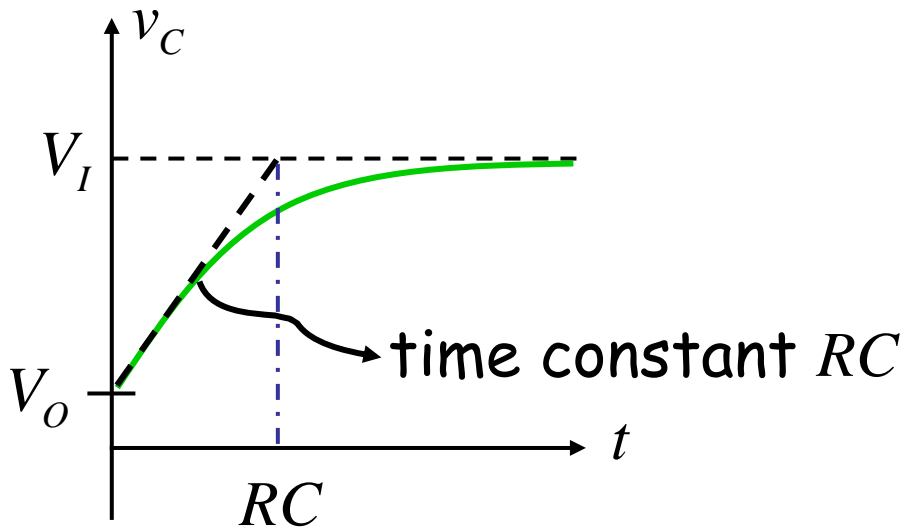
# Digital Circuit **Speed**

Cite as: Anant Agarwal and Jeffrey Lang, course materials for 6.002 Circuits and Electronics, Spring 2007. MIT OpenCourseWare (<http://ocw.mit.edu/>), Massachusetts Institute of Technology. Downloaded on [DD Month YYYY].

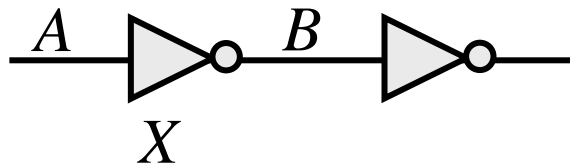
# Review



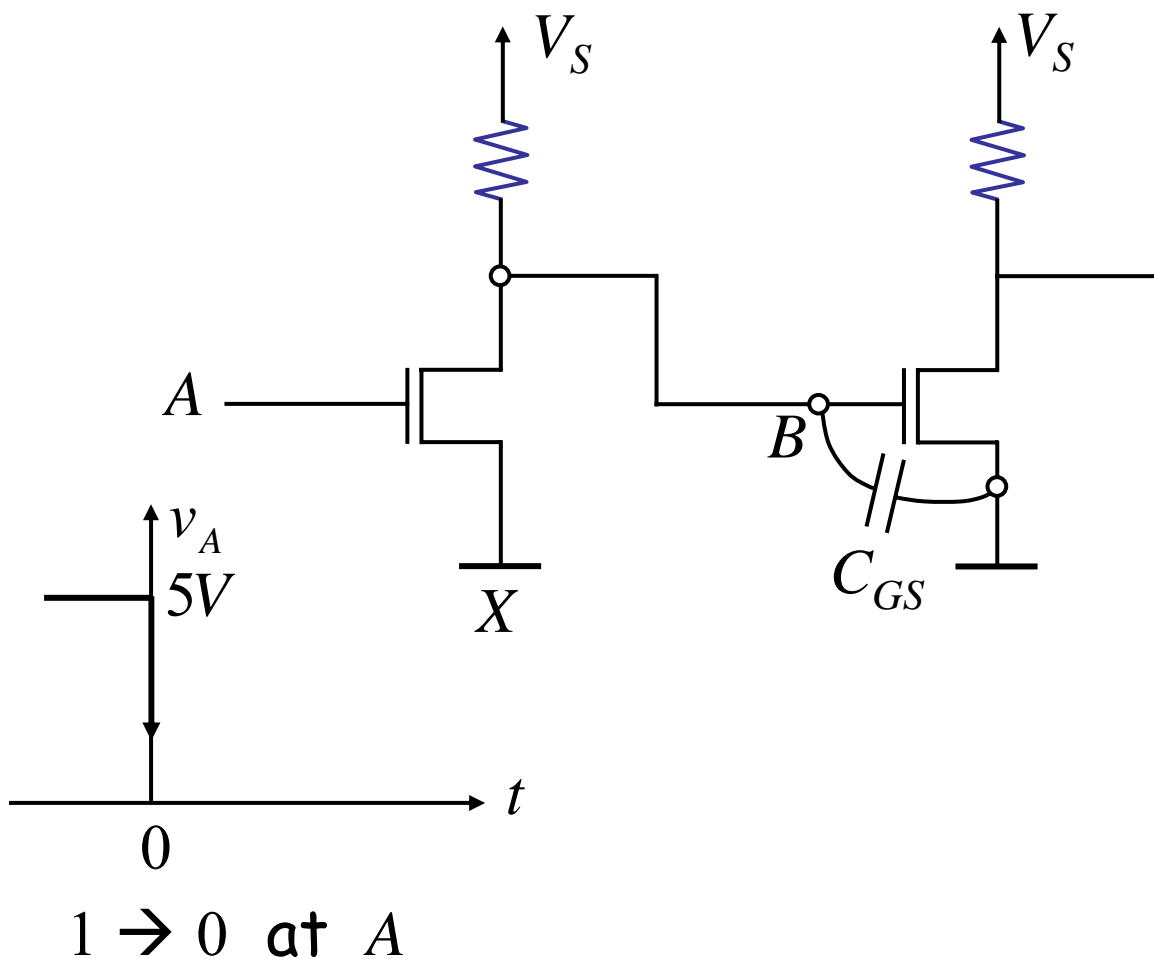
$$v_C = V_I + (V_O - V_I) e^{-\frac{t}{RC}} \quad \text{—————} \quad \textcircled{1}$$



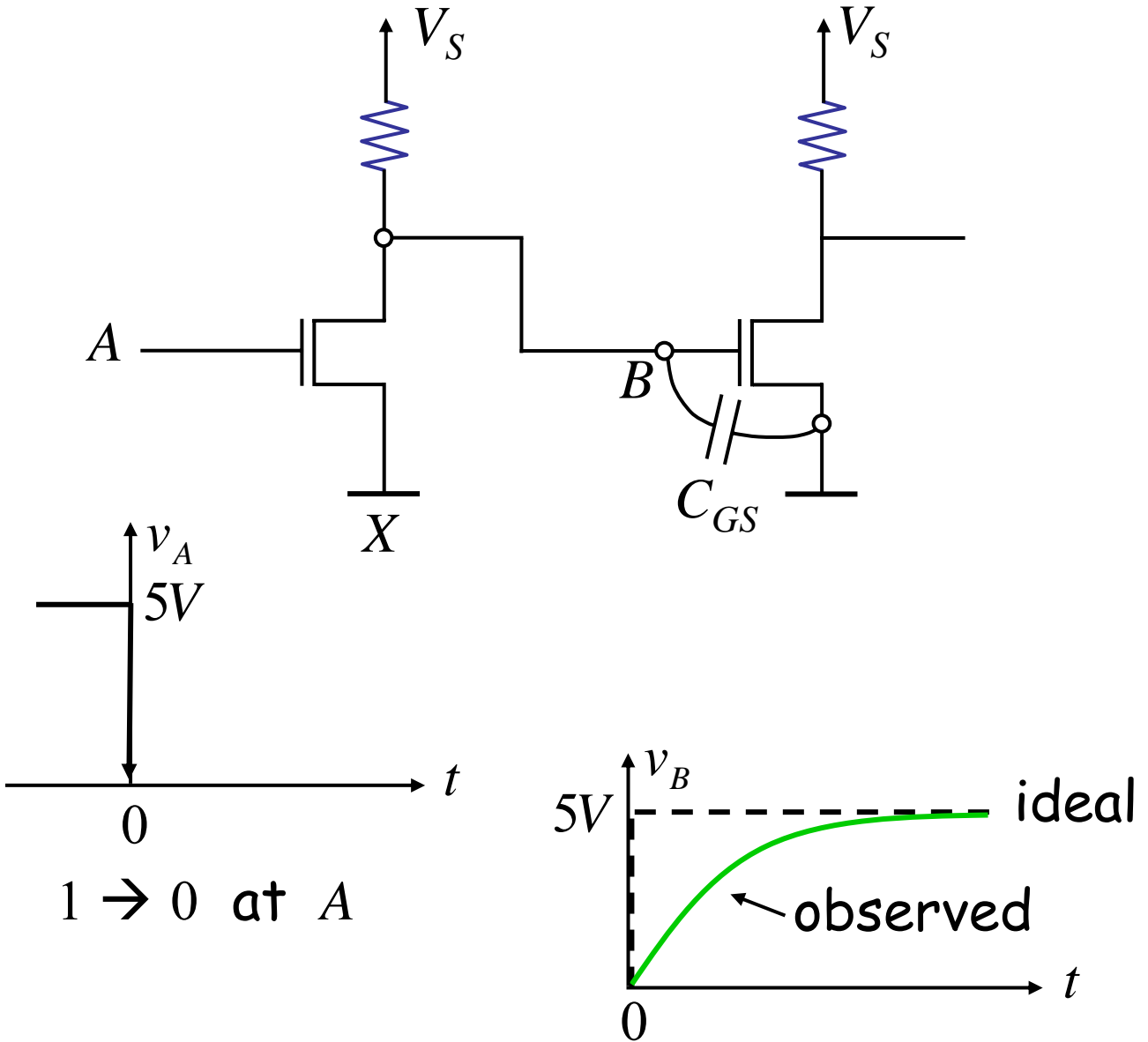
# Let's apply the result to an inverter.



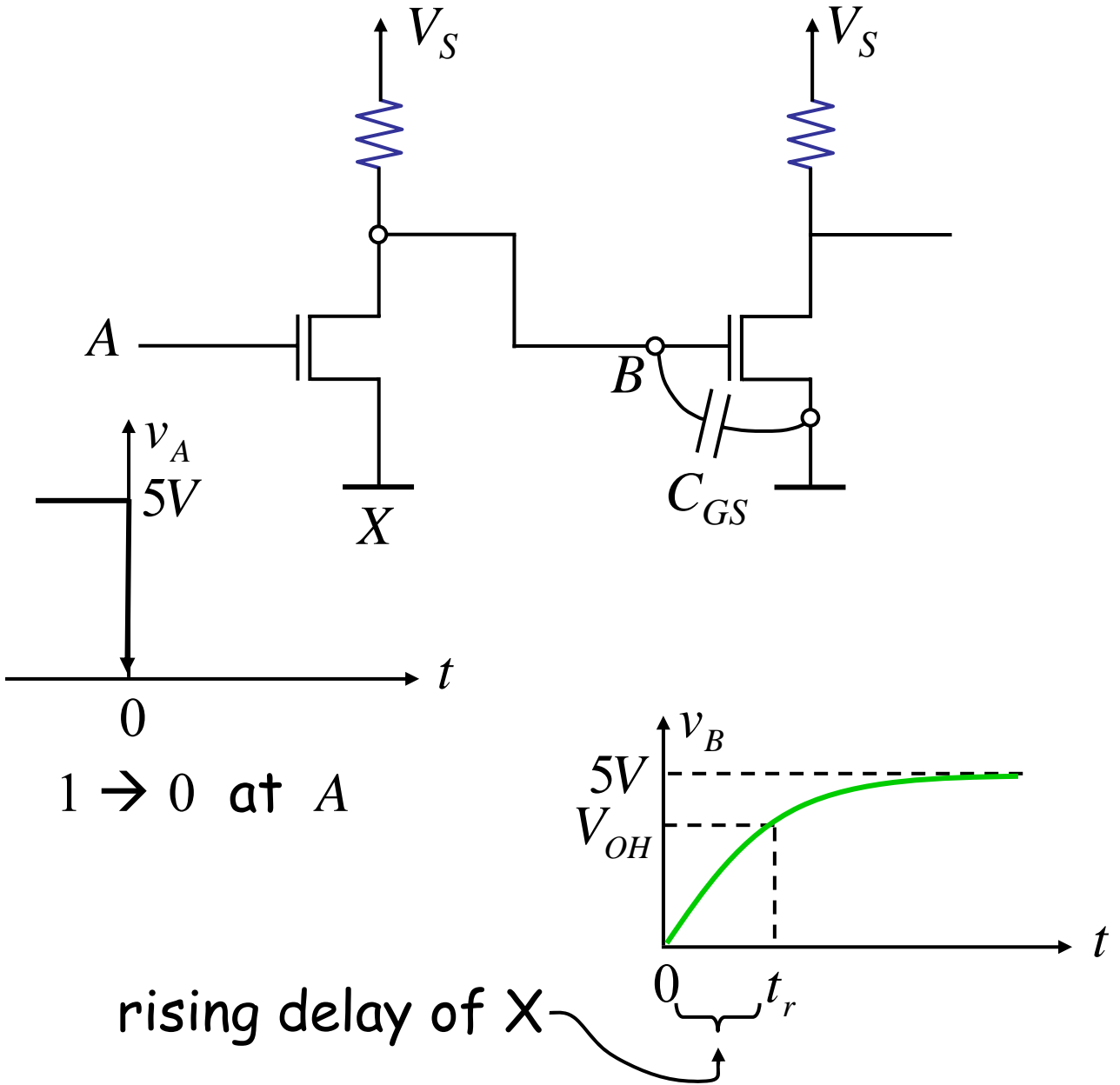
First, rising delay  $t_r$  at  $B$



# First, rising delay $t_r$ at $B$

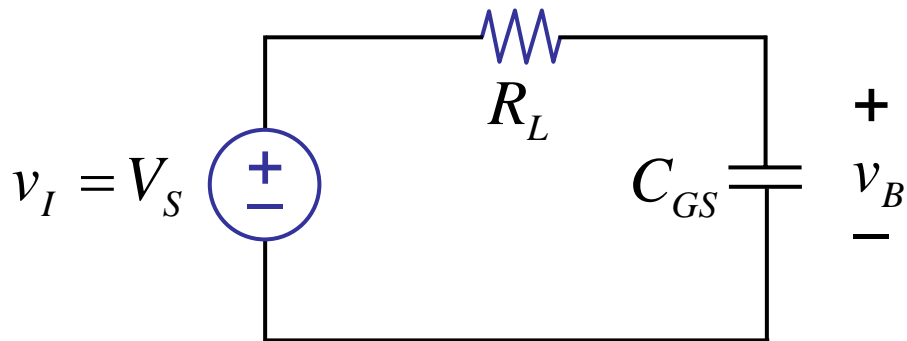


# First, rising delay $t_r$ at $B$



rising delay of  $X$

# Equivalent circuit for $0 \rightarrow 1$ at $B$



$$v_I = V_S$$
$$v_B(0) = 0 \quad \text{for } t \geq 0$$

From ①

$$v_B = V_S + (0 - V_S) e^{\frac{-t}{R_L C_{GS}}}$$

Now, we need to find  $t$  for which

$$v_B = V_{OH} .$$

Or

$$v_{OH} = V_S - V_S e^{\frac{-t}{R_L C_{GS}}}$$

Find  $t_r$  :

$$V_S e^{\frac{-t_r}{R_L C_{GS}}} = V_S - V_{OH}$$

$$\frac{-t_r}{R_L C_{GS}} = \ln \frac{V_S - V_{OH}}{V_S}$$

$$t_r = -R_L C_{GS} \ln \frac{V_S - V_{OH}}{V_S}$$

Or

$$v_{OH} = V_S - V_S e^{\frac{-t}{R_L C_{GS}}}$$

Find  $t_r$  :

$$V_S e^{\frac{-t_r}{R_L C_{GS}}} = V_S - V_{OH}$$

$$\frac{-t_r}{R_L C_{GS}} = \ln \frac{V_S - V_{OH}}{V_S}$$

$$t_r = -R_L C_{GS} \ln \frac{V_S - V_{OH}}{V_S}$$

e.g.

$$R_L = 1K$$

$$V_S = 5V$$

$$C_{GS} = 0.1 pF$$

$$V_{OH} = 4V$$

$$\begin{aligned} t_r &= -1 \times 10^3 \times 0.1 \times 10^{-12} \ln \frac{5-4}{5} \\ &= 0.16 ns \end{aligned}$$

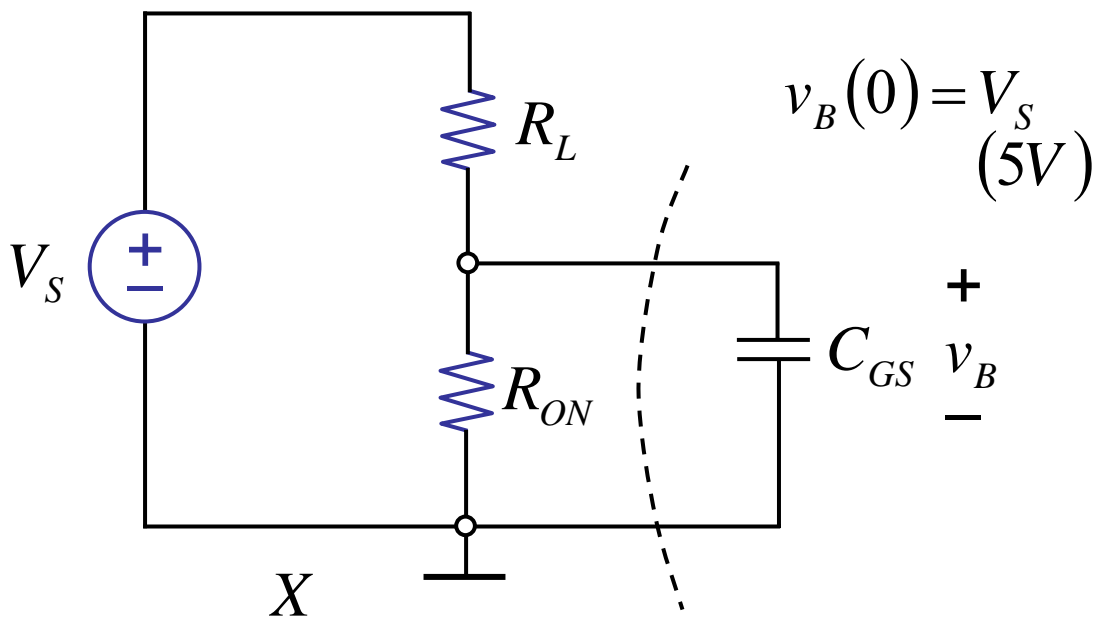
$$RC = 0.1 ns !$$



# Falling Delay $t_f$

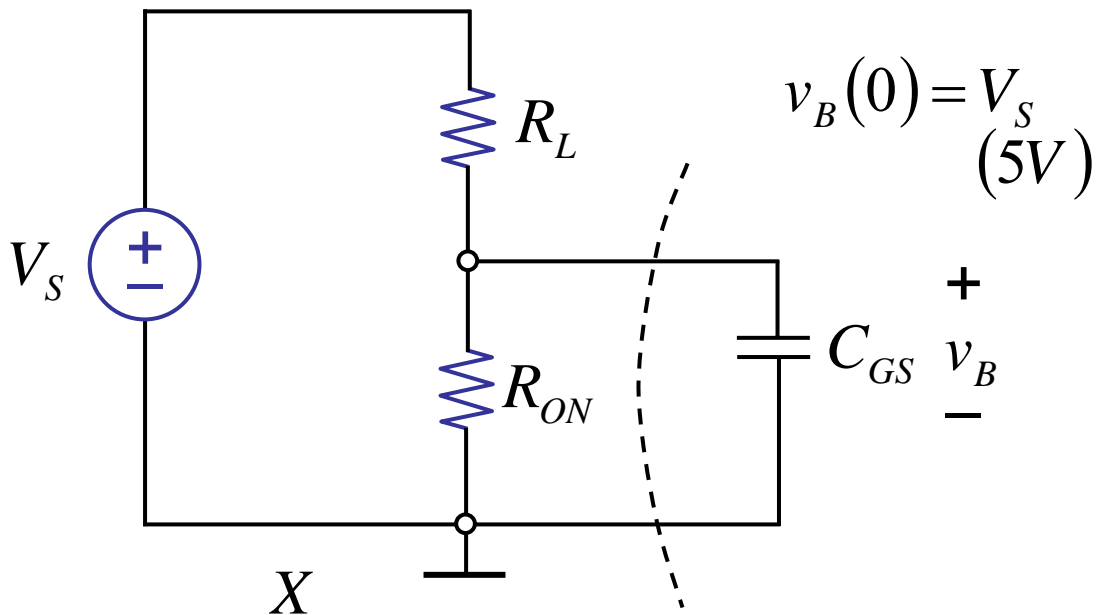
Falling delay  $t_f$  is the  $t$  for which  $v_B$  falls to  $V_{OL}$

Equivalent circuit for  $1 \rightarrow 0$  at  $B$

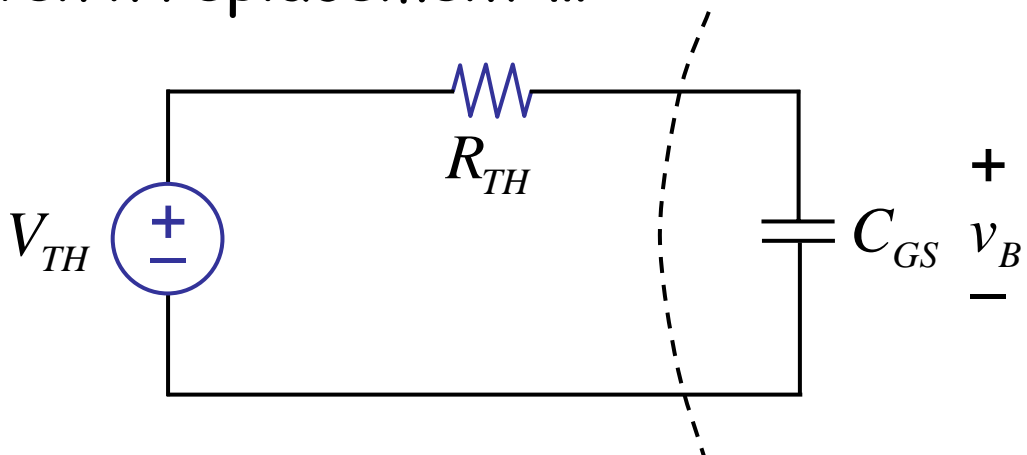


# Falling Delay $t_f$

Equivalent circuit for  $1 \rightarrow 0$  at  $B$



Thévenin replacement ...



$$R_{TH} = R_L \parallel R_{ON}$$

$$V_{TH} = V_S \frac{R_{ON}}{R_{ON} + R_L}$$

From ①

$$v_B = V_{TH} + (V_S - V_{TH}) e^{\frac{-t}{R_{TH}C_{GS}}}$$

Falling decay  $t_f$  is  
the  $t$  for which  $v_B$  falls to  $V_{OL}$

$$V_{OL} = V_{TH} + (V_S - V_{TH}) e^{\frac{-t_f}{R_{TH}C_{GS}}}$$

or

$$t_f = -R_{TH}C_{GS} \ln \frac{V_{OL} - V_{TH}}{V_S - V_{TH}}$$

$$t_f = -R_{TH} C_{GS} \ln \frac{V_{OL} - V_{TH}}{V_S - V_{TH}}$$

e.g.  $R_L = 1K$        $V_S = 5V$        $R_{ON} = 10\Omega$

$C_{GS} = 0.1 pF$        $V_{OL} = 1V$

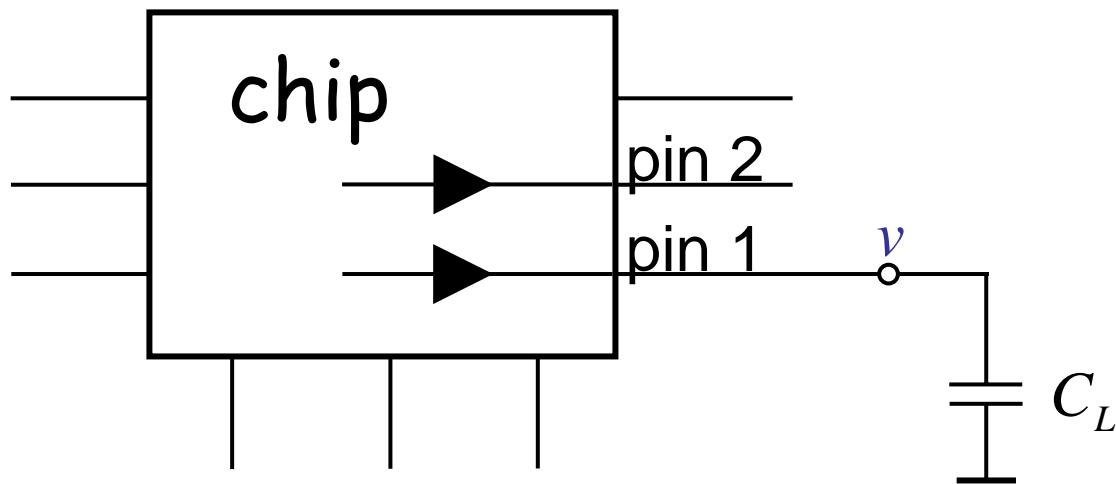
$R_{TH} \approx 10\Omega$ ,       $V_{TH} \approx 0V$

$$t_f = -10 \cdot 0.1 \cdot 10^{-12} \ln \frac{1}{5}$$

$$= 1.6 ps$$

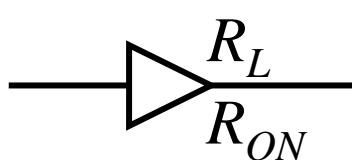
$$RC = 1 ps !$$

# For recitation: Slow may be better Problem



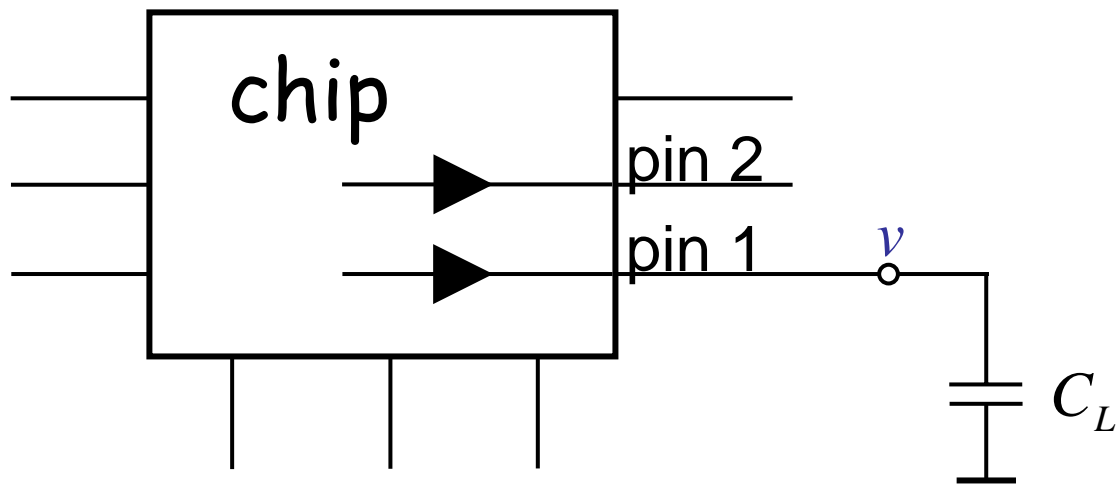
$v$ : ideal  observed  slow!

So the engineers decided to speed it up...

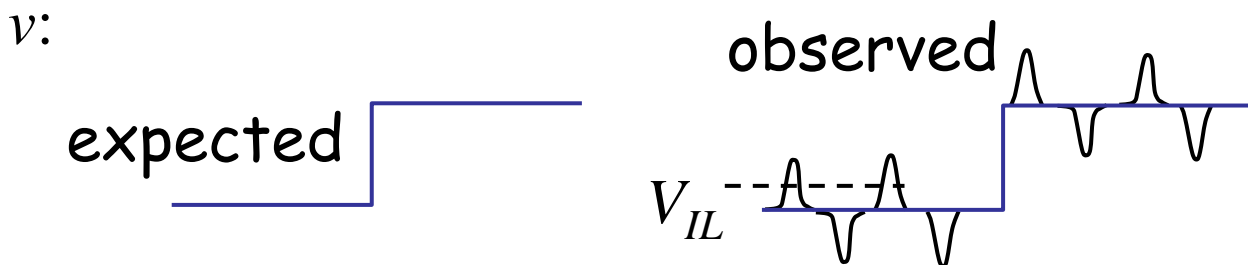


made  $R_L$  small  
made  $R_{ON}$  small

# For recitation: Slow may be better Problem

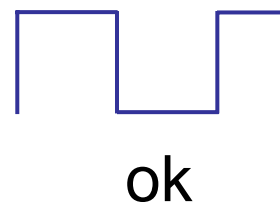
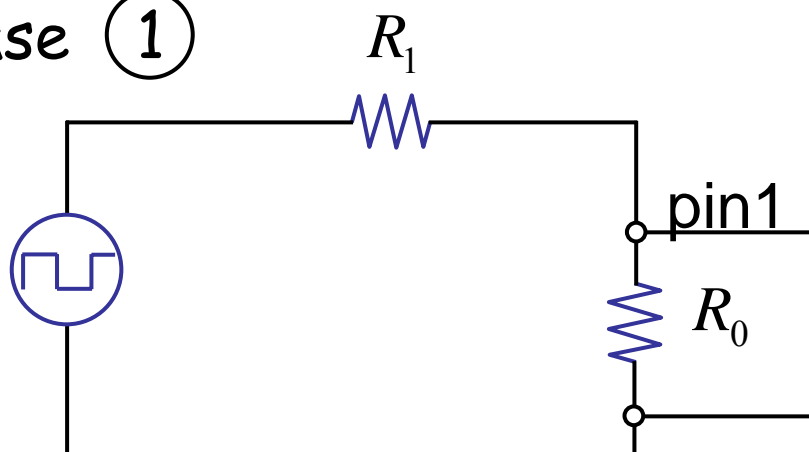


... but, disaster!



# Why? Consider ...

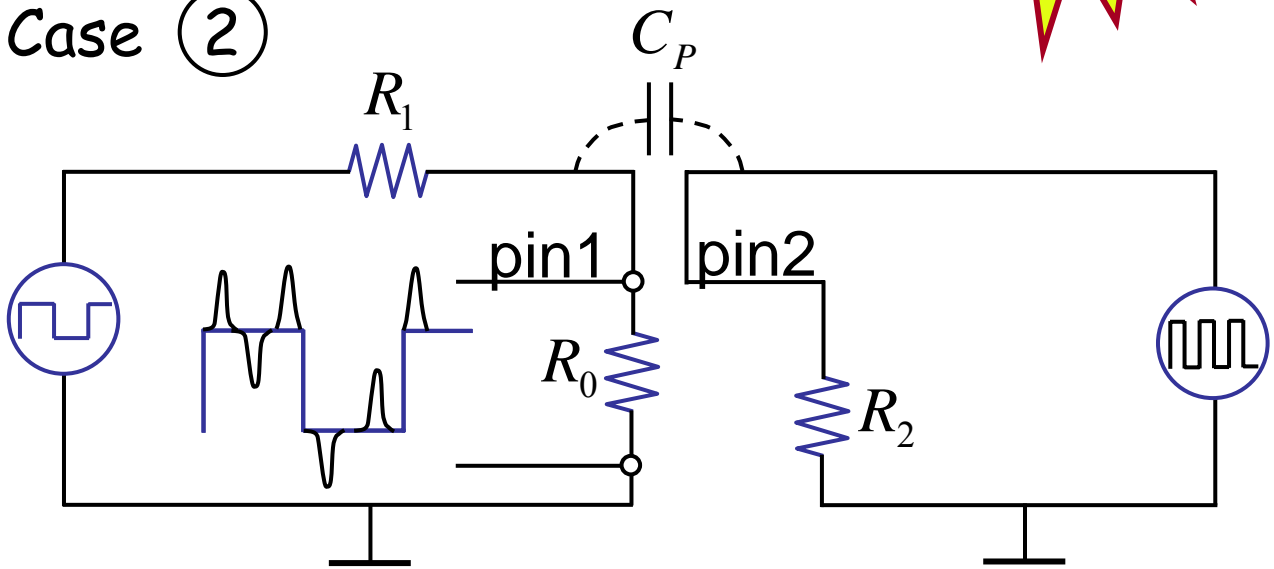
Case ①



# Why? Consider ...

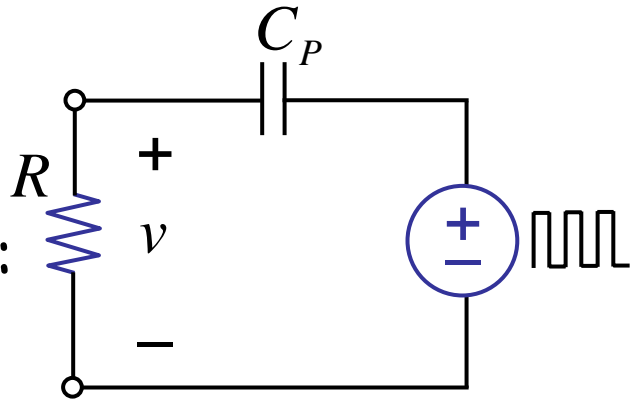


Case (2)



crosstalk!

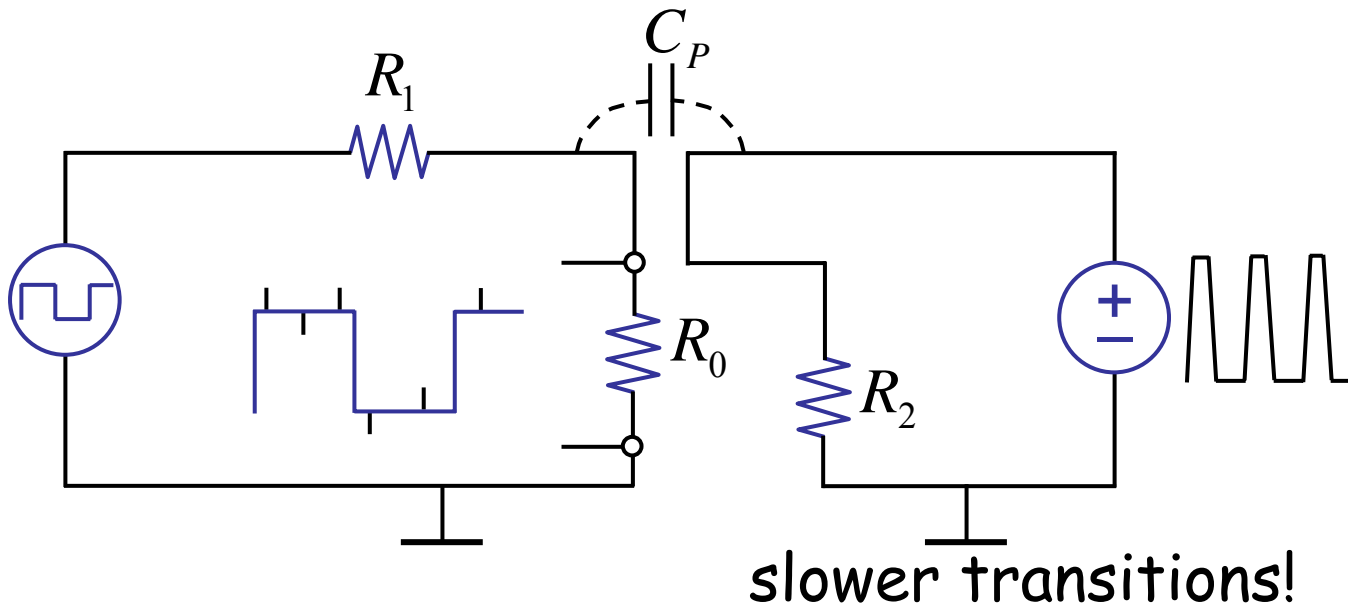
model for crosstalk:





## Case ③

... 6.002 expert saw the solution



Detailed analysis in recitation.