6.851 Advanced Data Structures (Spring'10)

Prof. Erik Demaine Dr. André Schulz TA: Aleksandar Zlateski

Problem 7 Sample Solutions

Finding the most significant 1 bit.

(a) We define $C_0 = 0^b$, and $(C_1) = 10^{b-1}$ $A_x = \left[x \;\texttt{|}\; (x + \texttt{~}C_1)\right] \;\texttt{\&}\; C_1$

Step-a(x)
1 $A_x \leftarrow x + 01^{b-1}$
2 $A_x \leftarrow A_x \;\texttt{|}\; x$
3 **return** $A_x \;\texttt{\&}\; 10^{b-1}$

(b) $B_x = \left((A_x \;\texttt{>>}\; (b-1)) \;\texttt{*}\; (0^b1)^{b-1} \;\texttt{>>}\; (w-b)\right) \;\texttt{\&}\; 1^b$

Steps-ab(x)
1 $A_x \leftarrow$ Step-a(x)
2 $B_x \leftarrow A_x \;\texttt{>>}\; (b-1)$
3 $B_x \leftarrow (B_x \;\texttt{*}\; [0^b1]^{b-1}) \;\texttt{>>}\; (w-b)$
4 **return** $B_x \;\texttt{\&}\; 1^b$

(c) Note that the order of the chunks has changed in part (b). We need to find the *least* significant 1 bit. Also, not that $-x = \texttt{~}x + 1$.
$C_x = b - (B_x \;\texttt{\&}\; -B_x)$

Steps-abc(x)
1 $B_x \leftarrow$ Step-b(x)
2 $C_x \leftarrow B_x \;\texttt{\&}\; -B_x$
3 **return** $b - C_x$

(d) Let $\alpha = \sum_{i=0}^{b-1}(1 \;\texttt{<<}\; i) \;\texttt{<<}\; i$, then by setting $y = (x \;\texttt{>>}\; C_x) \;\texttt{\&}\; 1^b$, and $z = (y \;\texttt{*}\; [0^{b-1}1]^b) \;\texttt{\&}\; \alpha$, we have reduced the problem to parts (a)-(c).

Full Algorithm

Most-Significant(x)
1 $C_x \leftarrow$ Steps-abc(x)
2 $y \leftarrow (x \;\texttt{>>}\; C_x) \;\texttt{\&}\; 1^b$
3 $z \leftarrow (y \;\texttt{*}\; [0^{b-1}1]^b) \;\texttt{\&}\; \alpha$
4 $C_z \leftarrow$ Steps-abc(z)
5 **return** $(C_x \;\texttt{<<}\; b + C_z)$

6.851 Advanced Data Structures

Spring 2010