

6.851 ADVANCED DATA STRUCTURES (SPRING'10)

Prof. Erik Demaine

Dr. André Schulz

TA: Aleksandar Zlateski

Problem 1 Sample Solutions

Transposing a matrix. Consider a point set $\{(x_i, i)\}$ of k^2 points on a $k^2 \times k^2$ lattice representing the access sequence. For each point (x_i, i) we introduce three new points at $(x_i - 1, i)$, $(k \lfloor \frac{x_i}{k} \rfloor, i)$ and $(k \lceil \frac{x_i}{k} \rceil, i)$.

The newly formed set is *Aborally Satisfied*, hence it represents a valid BST execution. The set contains $O(k^2)$, giving amortized cost of $O(1)$ per access.

Logarithmic redux. Consider the access sequence, the point set $X = \{(x_i, i)\}$ of m points on a $n \times m$ lattice. Let \hat{x} be the median of all $x \in X$. Inserting m points (\hat{x}, i) will ensure that each rectangle connecting a point left of or at \hat{x} and a point right of or at \hat{x} contains a point.

Now consider the two subsets of X , $X_{x \leq \hat{x}}$ and $X_{x \geq \hat{x}}$, each with at most m points, and at most $\frac{n}{2}$ distinct x values.

We recursively apply the same technique, to obtained point set that is *Aborally Satisfied*. We get the number of newly inserted points by solving the recursion $N(m, \frac{n}{2}) = 2N(m, \frac{n}{2}, n) + m$.

The total number of accesses is then $O(N(m, n) + m) = O(m \log n + m) = O(m \log n)$.

MIT OpenCourseWare
<http://ocw.mit.edu>

6.851 Advanced Data Structures

Spring 2010

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.